

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XLVII.

MAY 13, 1887

No. 7

J. W. L. GLAISHER, M.A., F.R.S, President, in the Chair.

W. Austin Ashe, Director of the Observatory, Quebec; and
Gustavus William Nicolls, Caixa 162, Pernambuco, Brazil,

were balloted for, and duly elected Fellows of the Society.

On the Images formed by Reflecting Mirrors, and their Aberration.
By the Hon. Lord McLaren.

The application of reflecting telescopes to photography renders it desirable that their suitability for large fields should be tested by an accurate investigation of the caustic images produced by the reflection of oblique pencils from their surfaces. This is the purpose of the present paper.

It has been suggested that for large fields a mirror of spherical figure possesses some advantages over that of the parabolic form, on the ground that the spherical mirror, while it does not bring any parallel pencil to an accurate focus, reflects always a circular image, and gives images of equal intensity at all angles. The present investigation shows that for fields of any angle likely to be required in practice—say for any angle under two degrees—the lateral aberration of the paraboloid is inconsiderable. But it also brings out a variation of focal length for different distances from the centre of the field, which even for a field of 2° is considerable. It will be shown in the sequel that the cycloidal mirror possesses some advantages in this respect over mirrors of the parabolic type.

The form of the caustic surface produced by direct reflexion from a spherical surface is well known: it is, near the image, a

cone generated by the revolution of an epicycloid, and presents its apex, or cusp, to the observer's eye. I have not found, however, in any work accessible to me, a solution of the problem of the caustic surfaces produced by the reflexion of oblique pencils from a paraboloid. Sir William Hamilton, in his great Memoir on the "Theory of Systems of Rays,"* finds expressions for the caustics formed by reflecting surfaces of any form. The distinguished author has not shown how these expressions are to be integrated for parabolic reflecting surfaces, and I am not aware that his differential equations have been integrated for the general equation of a surface of the second degree.

The solution which I offer for the case of a parabolic mirror is obtained in a different way, and is perhaps of some practical interest. It will be seen that for rays of any inclination to the axis of symmetry the caustic is a curve resembling a parabola, confocal with the reflecting surface, and having its axis inclined to that of the reflector at an angle depending on the inclination of the ray. The equation of the caustic curve, referred to its own axis, is

$$\rho^{\frac{1}{2}} \cos \frac{\theta}{3} = a^{\frac{1}{2}}.$$

The small arc of the caustic which constitutes the image of a non-central star is a diminished copy, or nearly so, of the arc of the reflector, but turned round so as to lie nearly in the line of sight. The image of the whole field is convex to the observer's eye, and is aplanatic only in this sense, that equal arcs of the heavens are represented by equal arcs in the image of the field.

CASE I.—*To find the equation to the caustic of the parabola for parallel rays in the principal plane.* (See Plate 7.)

Let $S_1S_2S_0$ be an axial section of the parabolic reflecting surface in the plane of reflection, O the focus and origin of polar coordinates.

S_1O , S_2O two consecutive vectors.

S_1P_1 , S_2P_2 two consecutive reflected rays, on which let the perpendiculars OP_1 , OP_2 be let fall from the origin.

S_0O (or X) is the axis of the parabolic reflecting surface.

S_0P_0 the reflected ray proceeding from the vertex, and making with the axis an angle $= (\phi)$.

OP_0 is the perpendicular on S_0P_0 from O .

Then the curve $P_1P_2P_0$ represents the locus of the intersections of the reflected rays with the perpendiculars drawn from O , and the curve $C_1P_0C_2$ represents the caustic, or locus of the intersections of consecutive reflected rays.

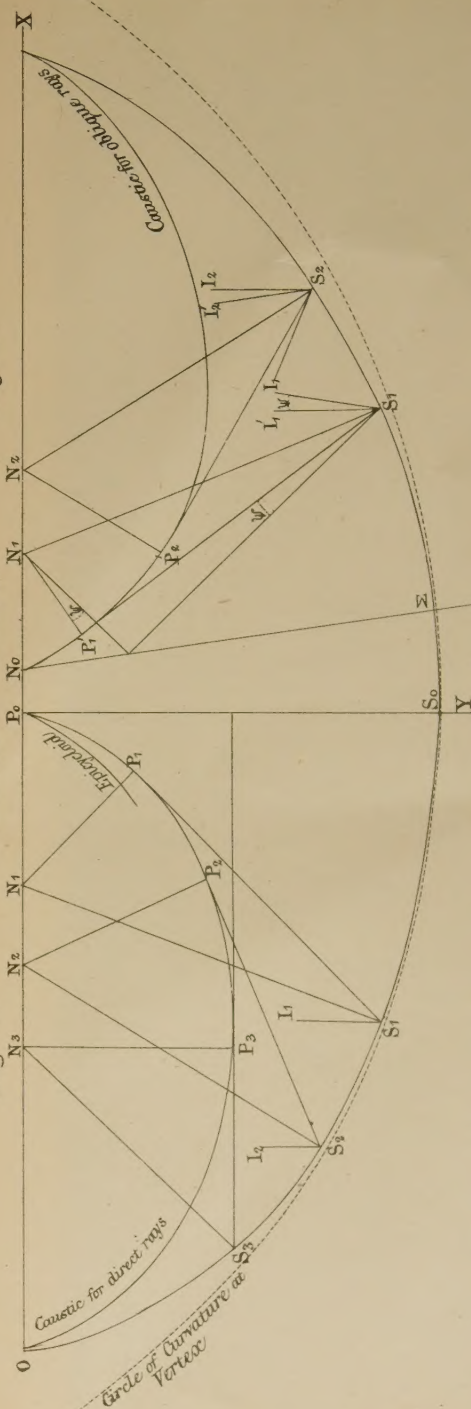
Let the axis of the reflecting surface be the reference line for the polar equation $f(\rho\theta)$ of the parabola, and also for the

* *Trans. Roy. Irish Acad.* Vol. XV.

The diagram illustrates the geometric construction of the caustic of a family of circles. Key elements include:

- Incident Rays:** A line labeled "Incident Rays" originates from point A .
- Mirror:** A line labeled "Mirror" represents the reflecting surface.
- Centers of Circles:** Points S_1 and S_2 are the centers of the circles in the family.
- Radii:** Lines r_1 and r_2 represent the radii of the circles centered at S_1 and S_2 respectively.
- Caustic:** A curve labeled "Caustic" is the envelope of the family of circles.
- Points on the Caustic:** Points P_0 , P_1 , and P_2 are marked on the caustic curve.
- Locus of Vertices:** A curve labeled "Locus of Vertices" is shown, passing through P_0 .
- Pedal Curve:** A curve labeled "Pedal" is also depicted.
- Other Points:** Points Q and O are marked on the diagram.
- Angles:** Angles ϕ and ψ are indicated at various points.

Fig. 2.



respective polar equations $f'(\rho' \theta')$ and $f''(\rho'' \theta'')$, of the curve $P_1P_2P_0$ and the curve $C_1P_0C_2$.

Then, because the incident pencil is a pencil of parallel rays, all the rays which are incident on the parabola at points S_1, S_2 , &c. are inclined at the same angle ($= \phi$) to the lines parallel to X which pass through these points; and all the reflected rays are inclined at the same angle ($= -\phi$) to the corresponding vectors.

Or, if (χ) be the angle between the normal and the x coordinate at any point S_1 ,

$(-\chi)$ the angle between the normal and the radius vector at the same point,

$(\chi + \phi)$ will be the angle of the incident ray at point S_1 ,

$-(\chi + \phi)$ will be the angle of the reflected ray at the same point.

In other words, all the reflected rays S_1P_1, S_2P_2 , &c. make equal angles ($= -\phi$) with the vectors passing through S_1, S_2 , &c.

Therefore, in the series of triangles OS_1P_1, OS_2P_2 , &c. all the angles at S_1, S_2 , &c. are $= \phi$;

and all the angles at P_1, P_2 , &c. are by construction right angles.

Therefore the remaining angles S_1OP_1, S_2OP_2 , &c. are each $= 90^\circ - \phi$.

If θ be the angle between the reference line and any vector, OS_1 of the parabola, then

$$\theta' = \theta + (90^\circ - \phi) *$$

will be the angle between the reference line and the corresponding vector OP_1 of the curve $P_1P_2P_0$. Also

$$\text{vector } OP_1 = \text{vector } OS_1 \times \sin \phi,$$

or

$$\rho' = \rho \sin \phi.$$

Now, if the reference line for the curve $P_1P_2P_0$ be turned round to OP_0 , while the reference line for the parabola $S_1S_2S_0$ is unchanged, then, for any correlative points S_1P_1 in the two curves whose current coordinates are $(\theta\rho)$ and $(\theta'\rho')$, we have

$$\theta' = \theta,$$

and

$$\rho' = \rho \sin \phi = \rho \times \text{constant}.$$

The curves are therefore similar in figure, and the curve $P_1P_2P_0$ is a parabola with parameter

$$4a' = 4a \sin \phi \quad \dots \dots \dots (a)$$

where $(4a)$ is the parameter of the reflecting curve.

* In the figure θ is negative, being in the fourth quadrant.

The equation of the curve $P_1P_2P_0$ may be thus written :

$$\rho^{\frac{1}{2}} \cos \frac{1}{2}\theta' = a^{\frac{1}{2}} \quad \dots \dots \dots (b)$$

From this construction the equation of the caustic may be immediately deduced. By construction the derived parabola, $P_1P_2P_0$, is the locus of intersections of perpendiculars drawn from O to the reflected rays. By definition the reflected rays are tangents to the caustic curve, $C_1P_0C_2$, and the caustic is thus the first negative pedal of the derived parabola.

The equation of the caustic is deduced as follows :

By a known property of pedal curves, if

$$\rho^m \cos m\theta' = a^m$$

be the equation to a curve,

$$\rho^{\frac{m}{m+1}} \cos \frac{m}{m+1}\theta' = a^{\frac{m}{m+1}}$$

is the equation of its first negative pedal.*

Hence, from the expression for the derived parabola [Equation (b)]

$$\rho'^{\frac{1}{2}} \cos \frac{1}{2}\theta' = a^{\frac{1}{2}}$$

we may at once write the equation of the caustic.

It is

$$\rho''^{\frac{1}{3}} \cos \frac{1}{3}\theta'' = a^{\frac{1}{3}} \quad \dots \dots \dots (c)$$

where a' is the prime vector in both curves, and $is = a \sin \phi$. The caustic touches the pedal, or derived parabola, at the vertex P_0 . For any value of θ'' not greatly exceeding 90° the caustic lies within the pedal parabola.

When $\theta'' = 270^\circ$, $\cos \frac{1}{3}\theta'' = 0$, and ρ'' becomes infinite.

The caustic is evidently a looped curve, with branches intersecting at the point where $\theta'' = 180^\circ$, and tending to ultimate parallelism with a line at right angles to its axis of symmetry.

Locus of Caustics.—If we suppose the inclination of the pencil to vary we shall have as many caustic curves as there are radiant points on the celestial arc, the quantity a' representing in each case the distance of the star image from the centre of the telescopic field. Before proceeding to the analysis for the surface of revolution, it may be noticed that in the parabolic mirror the locus of the vertices of the caustic curves is the point (P_0), being the angular point of the right-angled triangle OP_0S_0 . It therefore lies on a circle (shown in the diagram) whose diameter is the focal length, OS_0 , and radius $= \frac{1}{4}$ radius of curvature of the parabola at the vertex. That is to say, all the foci for pencils of different degrees of obliquity (or star-images of variable distances) lie on the circle; and all the

* Williamson, *Differential Calculus*, p. 231.

star-images in the field lie on a spherical surface whose radius $= \frac{1}{4}$ radius of curvature of parabola at vertex.

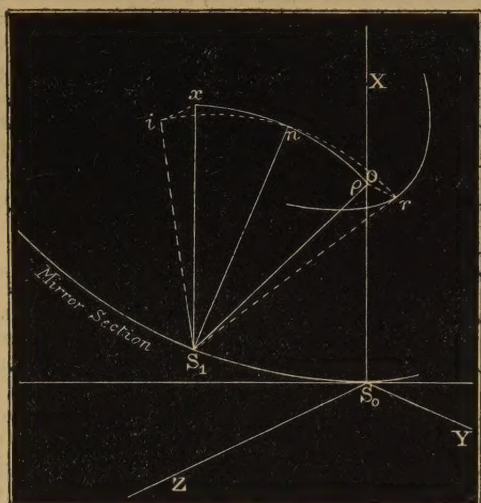


Fig. a.

CASE II. *Caustic Surface for the Paraboloid of Revolution.*—The caustic surface may be obtained by investigating the equation of the plane curve for any plane section of the paraboloid passing through the axis or at right angles to it.

Except in the case already considered, neither the incident nor the reflected ray lies in the plane of the section, nor do any two consecutive reflected rays lie in the same plane. The solution is therefore a little complicated, but it may be found by projective geometry commencing with the following preliminary proposition.*

The normal, the abscissa of the paraboloid, and the incident ray at any point S constitute a solid angle, whose six edges and angles are respectively equal to those of the solid angle formed by the normal, the vector, and the reflected ray. (See Figure a.)

If we denote these five lines by their initial letters we have

normal	= n	abscissa, parallel to X	= x
incident ray	= i	radius vector	= p
reflected ray	= r	S is the incident point.	

The same letters, n , x , i , r , and p , also denote the points of

* In what follows the reference plane for projections is a section through the axis of the paraboloid, and having any required inclination to the principal plane. It is assumed that the reflected rays intersect in some regular curve.

the two spherical triangles from which the angles at S are to be deduced.

To show that the solid angle i, n, x is equal to the solid angle $\rho n r$.—The three lines x, n, ρ are in one and the same plane, viz. the given axial section of the paraboloid, and angle $n S x = \text{angle } n S \rho$. Also the three lines $i n r$ are in one plane, being the plane of reflexion; and, by the law of reflexion, angle $i S n = \text{angle } n S r$. Now these equal angles are proportional to the corresponding sides of the two spherical triangles; also the contained angles formed by the intersection of the plane faces $i n r S, x n \rho S$ are equal. Therefore the remaining sides and angles are equal.

$$\therefore \angle r S \rho = \angle i S x = \phi \quad \dots \dots \dots (d)$$

$$\angle \text{ between faces } r S \rho, x S \rho = \angle \text{ between faces } i S x, x S \rho = \psi \quad \dots \dots (e)$$

These equations are fundamental. Equation (d) is independent of the particular section. It affirms that for any point S on the surface of a paraboloidal mirror the plane angle ($r S \rho$) between the reflected ray and the radius vector is *invariable*, and is equal to ϕ , the inclination of pencil to mirror axis.

In Equation (e) it is easily seen that the quantity ψ is the angle between a given axial section and the principal plane. This equation affirms that for any point S on the given mirror section the inclination of the plane faces $r_1 S_1 \rho_1, r_2 S_2 \rho_2$, &c. to the plane of the section is constant, and equal to ψ .

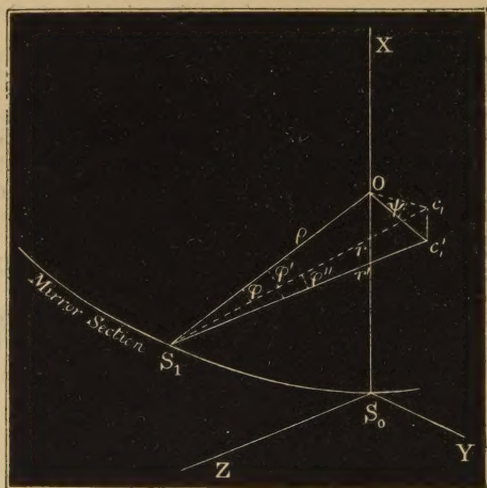


Fig. b.

From these equations it follows directly that at the variable point S the projection of the reflected ray on the plane of the section makes a constant angle (ϕ') with the radius vector, and

also that the projection of the reflected ray makes a constant angle with the reflected ray itself. (See Figure *b*, where c_1 is a point in the caustic, and c_1' a point in its projection on the plane of the axial section, OS_1c_1' .)

Because if we consider any of the solid angles formed at $S_1, S_2, \&c.$ by the vector, the reflected ray and the projection of the reflected ray (which may be denoted by r'), we see that its edges and angles are proportional to the angles and sides of a right-angled spherical triangle, of which one of the sides $= \phi$ and one of the angles (other than the right angle) $= \psi$. Now, as these quantities are the same for all the solid angles at $S_1, S_2, \&c.$ it follows that all the solid angles have their third edges and three angles respectively equal; or generally, for the solid angles at points $S_1, S_2, \&c.$ we have

Three edges respectively $= \psi, 90^\circ, 90^\circ - \psi,$

and three plane angles $\phi, \phi',$ and ϕ'' , whose equations are $\phi, = \angle$ between incident ray and x .

$$\tan \phi' = \tan \phi \cos \psi = \text{constant} \quad \dots \dots \dots (f)$$

$$\sin \phi'' = \sin \phi \sin \psi \quad \dots \dots \dots (g)$$

To form the equation of the caustic curve in the plane of projection.

With the same diagram and notation as in Case I., the plane of the paper is now the plane of the given axial section, making the angle (ψ) with the principal plane. The lines $AS_1, AS_2, \&c.$ are the projections of the parallel incident rays on the section. They are therefore also parallel, and make a constant angle $f(\phi\psi)$ with the corresponding *abscissæ*. $SP_1, SP_2, \&c.$ are the projections of the reflected rays, to which the perpendiculars $OP_1, OP_2, \&c.$ are drawn from the focus.

Then for any one of the points $S_1, S_2, \&c.$ we observe that the plane of projection intersects the plane of reflexion in the normal N ; and it is evident that equal angles measured from N , in the plane of reflexion, are represented by equal angles in the plane of projection. Therefore the projections of the incident and reflected rays make equal angles with the normal at any point, $S_1, S_2, \&c.$ of the series. Then, as before, $\rho_1\theta_1, \rho_1'\theta_1', \rho_1''\theta_1''$, are the current coordinates of the original and the two derived curves, whence we deduce the curve in the plane of projection identically as in Case I.

Angles $OS_1P_1, OS_2P_2, \&c. = \phi' [= \text{constant, by Equation } (f)].$

Angles $S_1P_1O_1, S_2P_2O_2, \&c.$ are by construction right angles. Therefore

Angles $S_1OP_1, S_2OP_2 = 90 - \phi' = \text{constant}.$

Hence $OP = OS \cdot \sin \phi'$, or

$$\rho' = \rho \sin \phi' \quad \quad \quad a' = a \sin \phi'$$

θ' (measured from its own axis OP_0) $= \theta$ (measured from x).

Hence the curve $P_0P_1P_2$, or derived parabola $f(\rho'\theta')$, is the locus of intersections of perpendiculars on the projections of the reflected rays.

Its parameter

$$4a' = 4a \sin \phi'.$$

And finally the negative pedal of the derived parabola is the projection of the caustic of the given section, or locus of intersections of the consecutive projections of the reflected rays.

It is as before

$$f(\rho''\theta'') = \rho''^{\frac{2}{3}} \cos \frac{\theta''}{3} - a'^{\frac{2}{3}} = 0,$$

where, approximately,

$$a' = a \sin \phi \cos \psi.$$

We thus see that for any axial plane of the paraboloid the projections of the reflected rays form a caustic curve, having the same equation as the caustic curve in the principal plane.

By projective geometry two curves which are connected by a projective relation are of the same degree and the same class, and differ only in their constant coefficients.*

We therefore conclude that the equation of the caustic is of the same form as the equation of its projection, and its parameter only remains for determination.

Caustic deduced from its Projection.—The reflected ray from the vertex of the mirror is evidently common to all the axial sections. It is therefore invariable in position and magnitude. The prime vector a of the caustic deduced in the principal plane is necessarily also common to all the axial sections; because it is the line drawn from the origin to the extremity of the ray reflected from the mirror vertex. That is to say, for all axial sections of the mirror we have caustic curves, of the same form and with parameter invariable in length and position; and, therefore, if the axial section of the mirror is turned round, so as to generate a surface of revolution, the caustic curve also will generate a surface of revolution about its axis of symmetry.

Circular Section of the Paraboloid.—The result just obtained may be found more easily by considering a circular section of the mirror in the plane YZ.

In such a section let ρ_1, ρ_2 , &c. be the vectors drawn from consecutive points S_1, S_2 , &c. of the circular section to the focus O. They are evidently of equal length, and their integral is a right cone, having its vertex at O. Let r_1, r_2 , &c. be consecutive reflected rays drawn from the same series of points S_1, S_2 , &c.

Then by Equation (d) (which is general for the whole reflecting surface) all the angles $\rho_1 S_1 r_1, \rho_2 S_2 r_2$, &c. (although lying in different planes) are equal to one another, being equal to the constant ϕ .

* Cremona, *Projective Geometry*, Chap. II.

Let perpendiculars OP_1, OP_2 be drawn from O to the reflected rays r_1, r_2 , &c. and we have

$$\begin{aligned} OP_1 &= \rho_1 \sin \phi; \\ OP_2 &= \rho_2 \sin \phi = OP_1 \dots \dots \dots (h) \end{aligned}$$

since all the vectors ρ_1, ρ_2 , &c. of the circular section are equal.

The locus of P accordingly is a small circle, described on the surface of a sphere whose radius $\rho' = \rho \sin \phi$.

This small circle is the caustic, because the consecutive reflected rays are tangents to it.

If we now consider two opposite points of the mirror section in the plane of the luminous pencil we see that for one of these the point P lies outside the cone of vectors, and for the other it lies inside the cone. The line joining the two points P is a diameter of the caustic circle; and by drawing the diagram in perspective for two parallel circular sections of the mirror it will be seen that the caustic circles are parallel and co-axial. Their integral is therefore a surface of revolution whose equation is that of the caustic curve in the principal plane.

Lateral Aberration.—In the caustic of the parabola we have not to find a circle of least aberration, because at the focus the aberration is *nil*, and we have only to determine for any particular point or image the diameter of a section of the cone of rays through the plane of the field of view. The aberration, whether in a direction radial or transversal to the field, is to be obtained from the equation to a section of the surface, in terms of the focal length (a), the semi-arc of mirror (θ), and the inclination of pencil (ϕ).

In the principal plane the required aberration is the difference of the distances of the *surface of caustic* and its tangent from centre of field, or focus of paraboloid.

The latter

$$= a'' \sec \theta, \text{ where } a'' = a \sin \phi.$$

The former

$$= \rho'' = \frac{a''}{\cos^3(\frac{1}{3}\theta)} = a'' \sec^3\left(\frac{\theta}{3}\right).$$

Therefore

$$\text{aberration} = \tilde{a} \sin \phi \left[\sec \theta - \sec^3\left(\frac{\theta}{3}\right) \right] \dots \dots \dots (k)$$

θ is determined by the aperture of mirror.

The rays proceeding from that half of the caustic surface which lies on the negative side of its prime vector constitute the right half of the pencil which enters the eye-piece; those proceeding from the positive side of the prime vector constitute the left half of the visual pencil. It is evident that near the centre of the field the images are symmetrical.

Except near the centre of the field there will obviously be a greater dispersion of the rays in the plane XZ than in

the plane XY of the incident pencil; and the diameter of the visible image will, apparently, be smaller in proportion to the dispersion.* If we assume that the visual pencil, in the direction XZ, is only effective to the extent of one-half of its angular divergence, then we have for this aberration the quantity

$$\psi = \sqrt{\frac{1}{2}},$$

and

$$a'' = a \sin \left(\frac{\phi}{\sqrt{2}} \right).$$

Therefore, by Equation (k), we have, in a direction transverse to radius of field,

$$\text{lateral aberration from centre of image} = a \sin \left(\frac{\phi}{\sqrt{2}} \right) \left[\sec \theta - \sec^3 \left(\frac{\theta}{3} \right) \right] \quad (l)$$

The breadth of the image will be twice this quantity.

Example of Computation of Lateral Aberration.

Assume focal length = 16 feet = 192 inches = (a) and diameter of mirror = 24 inches.

The semi-arc of mirror will be about $3^\circ 30' = (\theta)$

For a star 1° from centre of field, $1^\circ = (\phi)$

$$\log \sin \phi = 8.2419$$

$$\frac{\phi}{\sqrt{2}} - \frac{1^\circ}{1.4142} = 0^\circ.7072 = 43'$$

$$\log \sin \frac{\phi}{\sqrt{2}} = 8.0972$$

$$\sec \theta = \sec 3^\circ 30' = 1.0019$$

$$\sec \frac{1}{3}\theta = \sec 1^\circ 10' : \log \sec = .00009$$

$$\log \sec^3 \left(\frac{\theta}{3} \right) = .00027 \quad \left. \begin{array}{l} \sec \theta = 1.0019 \\ \sec \frac{1}{3}\theta = 1.0006 \end{array} \right\} \sec^3 \left(\frac{\theta}{3} \right) = 1.0006$$

$$\sec \theta - \sec^3 \left(\frac{\theta}{3} \right) = 0.0013 = f \text{ (secant).}$$

Then

(1) In the principal plane by Equation (k) we have

$$\log 192 \text{ inches} = 2.2833$$

$$\log \sin \phi = 8.2419$$

$$\log f \text{ (secant)} = \log 0.0013 = -3.1139$$

$$\log \text{lateral aberration} = -3.6391$$

$$\text{lateral aberration} = .0044 \text{ inch.}$$

* Since this paragraph was written the writer has found, by making a large scale perspective drawing of the course of the rays, considered as tangents to the caustic surface, that the converging pencils are apparently truly symmetrical cones, and that their images are therefore round. This agrees with the results obtained experimentally by Mr. Howard Grubb (*Monthly Notices* for April, p. 310); and it is evident from the equations that, if the images are circular near the centre of the field, they will be circular through its entire extent, provided the images are received on a concave plate of the proper curvature, as suggested in Mr. Grubb's paper.

(2) In the transverse plane by Equation (l) we have

$$\log 192 \quad 2.2833$$

$$\log \sin \frac{\phi}{\sqrt{2}} \quad 8.0972$$

$$\log 0.0013 \quad -3.1139$$

$$\log \text{aberration} = -3.4944$$

$$\text{transverse lateral aberration from centre of image} = \underline{\underline{.0031 \text{ inch}}}$$

$$\text{transverse diameter of image} = \underline{\underline{.0062 \text{ inch.}}}$$

III. *Elliptic and Hyperbolic Reflecting Surfaces.*

Although the writer has not been able as yet to obtain an exact solution of the caustic for parallel rays incident on an elliptic surface, it may be here mentioned that if, in the case of the ellipse, a similar figure to that of Case I. in this paper be drawn with divergent and convergent pencils, making a given angle with the focal pencils, the locus of intersections of perpendiculars on the reflected pencil will be a small ellipse placed obliquely to the primary ellipse, and the caustic will be the first negative pedal of the derived ellipse. The divergent pencil traced backwards will be found to proceed from a radiating surface conjugate with the caustic surface. The caustic curves thus derived do not differ sensibly from the caustic of the parabola when they are represented graphically, and from the manner of their derivation they are obviously curves involving the cubic radical. It would add needlessly to the bulk of this paper to give the demonstration for the case of the elliptic caustic. By drawing the diagram with divergent and convergent pencils, and using the same notation and analysis as in Case I., the reader will have no difficulty in finding that the caustic of the divergent pencils is the first negative pedal of an ellipse whose axis is inclined to the minor axis of the reflecting ellipse at the angle (ϕ) which the divergent pencil makes with the major axis. The same notation and analysis gives for the hyperbola, with converging incident pencils, and diverging reflected pencils, a caustic whose equation is the first negative pedal of a small hyperbola inclined to the reflecting hyperbola as in the case of the ellipse.

The relation found in Case I. between the caustic of the parabola and the equation of the parabola itself is thus shown to be only a limiting case of a more general theorem which includes all curves of the second degree. For if in the case of the ellipse, for example, the major axis be supposed infinite, then the converging pencil (whose rays are supposed to make a constant angle with those of a focal pencil) becomes a pencil of parallel rays having the given inclination to a pencil parallel to the axis of the reflecting surface.

IV. *Cycloidal Surfaces of Revolution*.—The investigation of the conditions necessary to the production of a perfect image by reflexion has led to the conclusion that these conditions are better fulfilled by the cycloid than by the parabola. This results from two well-known properties of this curve: (1) the base or line on which the generating circle rolls bisects all the radii of curvature; (2) equal linear intervals measured along the base from its centre correspond to equal angles between the radii of curvature.

From the *first* condition it results that, whatever may be the angular extent of the field of view, the geometrical foci of the images lie on the base-line of the cycloid or on the base-plane of its figure of revolution.

From the *second* condition it results that the field of view is a true stereographic projection of an area of the celestial sphere, equal intervals of arc in the sphere being represented by equal straight lines in the field of view.

Caustic of the Cycloid.—Consider first the case of a direct pencil parallel to the axis of symmetry of the mirror (represented in the left quadrant of the plate). The figure represents an axial section, the base being parallel to the axis of (x), and the axis of symmetry parallel to (y).

I_1S_1, I_2S_2 , &c. are the incident rays.

N_1S_1, N_2S_2 , &c. the normals.

P_1S_1, P_2S_2 , &c. the reflected rays.

$f(x, y)$ denotes the equation of the reflecting surface.

$f(x', y')$ the equation of the caustic.

$I_1S_1N_1, I_2S_2N_2$ are the angles which the normals make with lines parallel to the axis of x .

Then in the equation of the reflecting surface

$$I_1S_1N_1 - I_2S_2N_2 = \delta \tan^{-1} \left(\frac{dx}{dy} \right).$$

In the equation of the caustic $I_1S_1P_1, I_2S_2P_2$ are the angles which its tangents make with the axis of x . And because $I_1S_1P_1, I_2S_2P_2$ are the sums of the angles of incidence and reflexion

$$I_1S_1P_1 - I_2S_2P_2 = 2(I_1S_1N_1 - I_2S_2N_2),$$

or

$$\delta \tan^{-1} \left(\frac{dy'}{dx'} \right) = 2\delta \tan^{-1} \frac{dy}{dx} \dots \dots \dots (x)$$

That is, the curvature of the caustic, at P_1 , measured by the deflection of its tangent for unit increment of arc is double the curvature of the reflecting surface at the corresponding point S_1 , measured by the deflection of its normal for an equal increment of arc.

If ϕ_1, ϕ_2 be the angles between consecutive normals to the cycloid and the relative incident rays (parallel to y),

ϕ_1' ϕ_2' the angles between the same normals and the
base x

$$\phi_1 - \phi_2 = \phi_1' - \phi_2'.$$

If θ_1 θ_2 be angles between the incident rays and the reflected rays which are tangents to the caustic

$\theta_1' \theta_2'$, the angles made by the normals to the caustic with the base

$$\theta_1 - \theta_2 = \theta_1' - \theta_2'.$$

Therefore, from Equation (z), we have

$$\theta_1' - \theta_2' = 2(\phi_1' - \phi_2').$$

We may now directly compare the angles made by the normals of the two curves with the base-line. If we make the base-line the reference-line for angular measure, then at the point O , where the cycloid and the caustic are in contact, $\phi' = 0$, $\theta' = 0$, and for any other position the normals drawn from S and P to the respective curves make angles with the base-line, having the relation $\theta' = 2\phi'$.

If now we join N_1P_1 , N_2P_2 , &c. without making any assumption as to the value of the angles at P_1 , P_2 , &c. we have, by the general equation for reflecting surfaces:*

Length of reflected ray = $\frac{1}{2}$ radius of curvature $\times$ cosine of angle of reflexion,

or

$$SP = NS \cdot \cos NSP.$$

Therefore the angle at P is a right angle, and NP is a normal to the caustic. In other words, *the normals drawn to the respective curves from the extremities of the reflected ray intersect in the base.* Also

$$NP = NS \cdot \cos \angle SNP = NS \cdot \cos (\theta' - \phi') = NS \cos \phi',$$

which may be written :

$$\rho' = \rho \cos \phi' (ii)$$

Now consider ON, the intercept on the base-line made by the normals NS and NP at any point.

If $x = v$ be the length of ON, measured from O, and a the radius of the generating circle of the cycloid, we have by the "tangential equation" of the cycloid and trochoid

[illegible]

as an equation between coordinates of the reflecting surface (Williamson, *Diff. Calc.* p. 339).

* In the Memoir by Sir William Hamilton, before referred to, this relation is deduced from the general differential equation of the reflected ray, and is shown to be true for any reflecting surface, where the ray is contained in the plane of the greatest or least osculating circle to the mirror (*Trans. Roy. Irish Acad.* Vol. XV. p. 97).

And in the equation of the caustic we have, for the same values of ON or ν ,

$$\theta' = 2\phi' \therefore \nu = a\theta' = 2a'\theta'. \quad (c)$$

The caustic, therefore, satisfies the "tangential equation" for the cycloid and trochoid generated by a circle of radius $a' = 2a$. It must be a cycloid if its normal vanishes at two points on the base-line. At the point O, where the reflecting curve and the caustic are in contact

$$SN \text{ or } \rho = 0 : \rho' = 0 \text{ [by Equation (a)]}.$$

At the centre of base-line

$$\rho = 2a, \phi' = \frac{\pi}{2}, \rho' = \rho \cos \phi' = 0.$$

The caustic meets the base in two cusps, one of them being coincident with the cusp of the reflecting curve, and the other being at the middle of the base-line.

The caustic is therefore a cycloid with radius of generating circle:

$$a' = \frac{a}{2} \quad (d)$$

and base

$$2\pi a' = \frac{1}{2} \text{ base of reflecting cycloid} \quad (e)$$

The whole caustic surface is generated by the revolution of the caustic-section about the axis of symmetry.

In connection with the caustic of oblique pencils we shall have occasion to refer to the simultaneous equations (a) and (c):

$$\rho' = \rho \cos \phi' \quad \nu = 2a'\theta'.$$

Caustic of Oblique Pencils.

With similar notation and construction (see the right quadrant of the plate), and denoting by (ψ) the inclination of the obliquely incident rays to the axis of symmetry of the reflector, and by Σ the point where the oblique pencil is normal to the surface, we have, as before, from the general equation of reflecting surfaces,

$$SP' = NS \cdot \cos NSP',$$

and from the preceding analysis we have: angle at P, a right angle: NP', a normal to the caustic, intersecting NS, the normal to the reflecting surface at a point N in the base-line. Now if we imagine a direct ray and an oblique ray, both incident on the reflector at the same point S, so that the construction in figure (1) is superposed on that of figure (2), we see that the normal SN and the point N are common to both systems. We see

also that the reflected rays SP and SP' (of direct and oblique reflexion) are inclined to each other at the angle ψ ; also that the normals PN and P'N (of the caustics of direct and oblique reflexion) have the same inclination $=\psi$. Hence for any one point in the base-line,

$$\theta'' = \theta' \mp \psi;$$

and for *any* two points in the base-line, $(x = \nu_1) \quad (x = \nu_2)$

we have, by subtraction,

$$\theta_1'' - \theta_2'' = \theta_1' - \theta_2'; \quad \nu_1' - \nu_2' = \nu_1 - \nu_2;$$

that is, for equal differences of base (or progression of the rolling circle) the differences of deflection of the normals in the two caustics are equal.

The caustic of oblique reflexion is therefore a cycloid satisfying an equation of the form

$$\nu = a''\theta''.$$

It consists of two unequal branches meeting in the focus of the oblique pencil, or rather of two separate cycloids on unequal bases, and together occupying the base of the reflecting cycloid.

Observing that the length of base of the caustic of direct reflexion is

$$\nu = 2\pi a',$$

it is easily seen that for the caustic of oblique reflexion the bases of the two branches are

$$2\pi a'' = 2a'(\pi + \psi) \quad \dots \dots \dots (f)$$

and

$$2\pi a''' = 2a'(\pi - \psi) \quad \dots \dots \dots (g)$$

by which the generating radii a'' and a''' are determined.

It is supposed that in the case of the oblique pencil the whole caustic surface is formed by revolution with varying parameter about the line ΣN , which is thus an interior tangent to the surface. For direct pencils the caustic surface is obviously a true figure of revolution about S_0P_0 .

The result of a careful investigation of the aberration is that at the circles of least aberration or least confusion there is very little difference between the sphere and the cycloid in the matter of lateral aberration, while if the preceding analysis be well founded there would be little or no longitudinal aberration for the images formed in a cycloidal mirror. The epicycloidal caustic of the circle of curvature falls within the cycloidal caustic in the figure.

The limits of this paper, however, do not admit of the further discussion of this subject.

Results.

The chief interest of the discussion turns on the possibility of obtaining symmetrical images from the caustics of oblique pencils of parallel rays.

1. In every caustic there are two principal points in the curve to be considered. One of these (usually denoted by the letter q) is the focus of the elementary pencil reflected from the central point of the mirror. The other principal point is the focus of that particular element of the pencil which is a normal to the reflecting surface (F).

In astronomical instruments, where the celestial arc viewed in the field is of less angular extent than the arc of the mirror, either of the points described is a possible focus. The question which of the two is the true visible focus of the infinitely distant point is one which must be considered separately for each kind of reflecting surface; because, in general, a visible image will only be formed at or near the point where the curvature of the caustic is a maximum, and the point of maximum curvature can only be determined from the equation of the caustic itself.

2. Other conditions being supposed equal, that form of mirror is to be preferred in which the visible image is formed at the point q ; that is to say, at the focus of the element of the converging pencil which comes from the centre of the mirror.

It is proved in the preceding paper that for a paraboloidal mirror this point constitutes the *vertex* of the cubic radical, which is the caustic. At the vertex $\frac{ds}{dp}$ is a maximum; and a greater number of tangent rays will pass through a small finite arc of the caustic at the vertex than will pass through a consecutive arc of equal length. Near this point is the visible image, and it is a remarkable and apparently unique property of the paraboloid that all its visible images are formed *by the rays proceeding from the central part of the mirror*. Moreover the image point is not a cusp, but only a point of maximum intensity in the luminous surface.

It is possible that there is also a secondary focus at the point F where the oblique pencil is normal to the parabola; but this focus has not been investigated.

3. It has been shown for the case of the PARABOLOID that the locus of the field of view is a sphere touching the paraboloid at the vertex, and whose radius is one-fourth the radius of curvature at that point; and, generally, whatever be the nature of the reflecting surface (provided it is symmetrical about a centre), the sphere here found will be the locus of the focus of rays incident near the centre of the mirror. I am indebted to Professor Cayley for the following proof of this interesting theorem:—

In ANY small mirror for parallel rays inclined to the axis in

the principal plane, the locus of the focus is a circle whose diameter $= \frac{1}{2}$ radius of curvature at vertex. (See Figure c.)

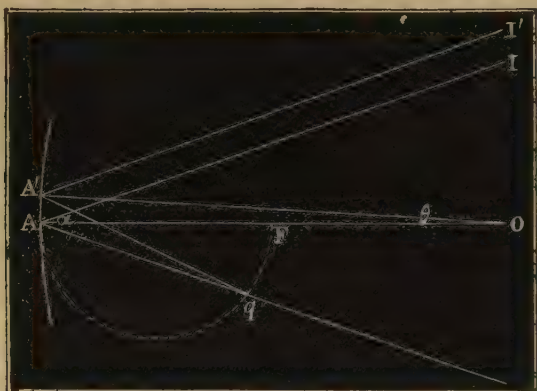


Fig. c.

a the radius of curvature OA .

α the inclination IAO .

θ the small angle $A'OA$.

The inclination of $A'q$ to axis $= \alpha + 2\theta$; whence if xy are coordinates of A' , and XY current coordinates,

Equation of

$$A'q \text{ is } Y - y = -(X - x) \tan(\alpha + 2\theta)$$

$$Aq \text{ is } Y = -X \tan \alpha$$

$$x = a(1 - \cos \theta) = -\frac{1}{2}a\theta^2$$

$$y = a \sin \theta = a\theta.$$

Equations are

$$Y - y = -(X - x)(\tan \alpha + 2\theta \sec^2 \alpha)$$

say

$$Y + X \tan \alpha = a\theta - 2X \sec^2 \alpha \cdot \theta$$

$$Y + X \tan \alpha = 0$$

whence

$$a\theta - 2X \sec^2 \alpha \cdot \theta = 0, \text{ i.e. } X = \frac{1}{2}a \cos^2 \alpha; Y' = -\frac{1}{2}a \sin \alpha \cos \alpha$$

$$X^2 + Y^2 = \frac{1}{4}a^2 \cos^2 \alpha = \frac{1}{2}aX,$$

which is circle on diameter AF (if $AF = \frac{1}{2}AO$).

In the case of the paraboloidal mirror the unsymmetrical form of the visible images near the boundary of the field is attributable mainly to the curvature of the field-surface, which has of course the effect of throwing the non-central part of the field outside the focus of the eye-piece or outside the plane of the

photographic plate, as the case may be. With a photographic plate ground to a *concave* surface of the proper curvature, there is reason to believe that symmetrical star images may be obtained in a field of the usual angular extent. But this is subject to the objection that the plate will always stop out the central and best part of the pencil.

4. In the SPHERICAL MIRROR the visible image will not coincide with the focus for rays incident at the centre of the mirror unless the angle subtended by the arc of the mirror is supposed to be small relatively to the angle of incidence of the oblique pencil. For the actual conditions of astronomical images the visible field will be the locus of the point F, or locus of the epicycloidal cusps, which are the points of greatest condensation for the respective pencils. For any given pencil the cusp of the caustic lies in the line where the ray is a radius of the spherical surface. Hence the geometric field is a spherical surface concentric with the mirror and having half the radius; and the visible field lies very near it.

5. Evidently a spherical mirror will not give symmetrical images, except near the centre, because it is only at the centre of the field that the visible images are formed by the central part of the mirror. Neither will the sphere give a flat field, though it will give a nearer approach to one than the parabola. Against this advantage we must set off the objection that, except at the centre of the field, its images are not formed by the central part of the mirror.

6. As to the CYCLOIDAL MIRROR, if the preceding analysis be correct, the locus of the cusps of the cycloidal caustic is a plane surface, but the images become unsymmetrical as they recede from the centre of the field. It is not meant to be asserted that the visible field will be a true plane, because the visible field is the locus of the circle of least aberration, which is not identical with the locus of the caustic cusps. But the field will be very nearly a plane surface.

It is right to add that the possibility of obtaining a flat field by reflexion has been doubted by mathematicians of high authority, and it must be left to the reader to form his own opinion of the validity of the reasoning by which the conclusion last announced has been reached.

On some Nebulæ hitherto suspected of Variability or Proper Motion.
By J. L. E. Dreyer, Ph.D.

The discrepancies met with in comparing observations of nebulae by different observers are frequently so great that more or less positive assertions have naturally from time to time been made as to variability or changes in the objects. While many of these assertions can at once be dismissed as showing that too

little regard has been paid to the great difficulties attending observations of nebulae or to the vast influence of the state of our atmosphere from night to night, others are either founded on undoubted facts or appear at least to others than practised observers of nebulae to be so. It seems that the only well authenticated cases of change in nebulae are changes of brightness only, while we *so far* do not possess any clear evidence of change of form or change of place.

The most generally known case of a nebula having disappeared—that of Hind's nebula in *Taurus*—is also the most certain and undisputed one. Chacornac's nebula (G.C. 1191), though only seen by that observer, doubtless also existed in a place where no nebulosity has since been seen; but these two cases are the only ones which are quite certain. It is true that some of William Herschel's nebulae cannot now be found, but these may either have been comets, or, more probably, some error of observation has vitiated the position he gives for the object in question. The latter case must, *e.g.*, have occurred on April 2, 1801, when he compared a number of nebulae (among which are three of the first class) with one comparison star, which he identifies as “208 (N) *Camelop.* of Bode's Cat.” As not one of these objects can now be found, it is evident that he made a mistake either in identifying the star or in making or recording the observation of it. Another case is G.C. 2179 = I. 26, which possibly may be an erroneous observation of the neighbouring M 95. The remarkable nebula at Merope, and a less notable one, G.C. 710, were by d'Arrest and others supposed to be variable, because the difficulty of seeing them with a large aperture made it appear strange that they had been described as conspicuous in small instruments. This difficulty is, however, now universally understood to arise from the use of too high a power with consequent smallness of field; and nobody now suspects these two nebulae of variability.

I shall not enter into an examination of the numerous instances where observers disagree as to the brightness of an object. Probably atmospheric or instrumental circumstances could in most cases account for this disagreement; but, all the same, the possibility of nebulae changing in lustre cannot be denied, since we have witnessed the total disappearance of two of them. But I propose to go through all the objects which have been suspected of having changed in form or position, and I trust I shall be able to prove that not one of the cases can be considered as well established.

Most of the nebulae examined in the following are double nebulae, which have been suspected of being in motion. They have been collected (from d'Arrest's work) by M. Flammarion in an appendix to his “Catalogue des Étoiles doubles et multiples en mouvement relatif certain.” In all cases the suspicion is founded on differences in their relative positions, as recorded by Sir William and Sir John Herschel and by later

observers, and it does not seem to have been generally noticed that Sir W. Herschel never employed a micrometer, but merely *estimated* the position-angles and distances of neighbouring nebulae, and that Sir John Herschel did the same from the commencement of his observations, and up to July 5, 1828.* After this date the eye-piece of his 18-inch Reflector was furnished with a wire micrometer, with which, however, only position-angles could be measured, while, as formerly, distances (both of nebulae and double stars) were only estimated. Even then he frequently estimated angles, in which case they were given only to whole degrees; whenever a decimal is given it shows that the position-angle was measured. Although such estimations should evidently not be used (or at least only with great caution) in drawing conclusions as to orbital or proper motion, I have thought it desirable to re-observe many of the objects with the new 10-inch Refractor at the Armagh Observatory, which I am devoting to micrometric observations of nebulae. The pair h 444-445 were measured with a filar micrometer; the others with a small micrometer with steel bars instead of spider lines, and requiring no illumination. Although distances measured with the latter kind of micrometer are doubtless subject to systematic errors (which I intend to investigate), their accuracy is quite sufficient for the present purpose.

Great Nebula in Andromeda.—The question as to variability of this object, first raised by Le Gentil, has been thoroughly discussed by G. P. Bond in his well-known Memoir, and he comes to the conclusion that the views of Le Gentil are “far from being supported by an amount of evidence adequate to such a conclusion.” The nucleus has been drawn or described in a remarkably different manner; by some (*e.g.* Schultz, Schönfeld, Vogel) as starlike, by others at the very same time (*e.g.* Schmidt: see Vogel’s note in *Astr. Nachr.* No. 2681) as a very soft and gradual condensation. These strange discrepancies are, however, explained by the valuable experiments made by Dr. Copeland† with different eyepieces, which show what an immense influence the magnifying power has on the appearance of the nucleus, the lower powers making it more starlike, the higher ones more soft-looking and extensive. Whether the new star of 1885 really belonged to the nebula or not does not concern us here; but Dr. Copeland’s experiments with artificial stars and different illumination of the field prove that, even if a real change had taken place in the nebula at the time of the outburst, it could not have been detected as long as the star was shining brightly. Since the fading of the star the nebula has quite resumed its former appearance.

III. 228-229 = h 251-252.—M. Flammarion remarks that W. Herschel estimates the distance at about 1', while d'Arrest gives

* See *Mem. R.A.S.*, vol. iv. p. 331.

† *Monthly Notices*, vol. xlvii. p. 60.

$\Delta\alpha = 8^s$, and estimated (on one occasion) the distance = $112'' \pm$. There is, however, perfect accordance between J. Herschel and d'Arrest, for the former gives $\Delta\alpha = 7^s.5$, while the distance resulting from his positions is $134''$. Both nebulae are very faint, especially the following one, which d'Arrest describes as "*indubitatum nebulae vestigium, adeo tenue tamen, ut visum ægre sustineat.*" His estimated distance is too small, as one can see by a glance at his $\Delta\alpha$.

III. $574-575 = h\ 294-295$.—While Sir W. Herschel does not say anything about the relative positions of these two very faint objects, Sir John gives the following positions for 1830 (one observation):—

$h\ 294$	3	10	25.8	49	$1822''$	{ The <i>np</i> of 2. Pos. from the following, which is the largest, $352^\circ.4$, dist. $100'$.
295	3	10	26.9	49	1644 .	

The *nf* of two.

while according to d'Arrest $\Delta\alpha = 4^s$, the following one being $124''$ south of the preceding one. A glance at Sir J. Herschel's positions and remarks shows that no conclusion can be drawn from them. If the first one was "the *np* of 2," the second cannot have been "the *nf* of 2," and if the position-angle of the first one from the second one was $352^\circ.4$, the N.P.D. of the first one must have been smaller than that of the second, and not greater. D'Arrest's $\Delta\alpha$ and $\Delta\delta$ give the position-angle of the first one = $339^\circ.9$ and the distance = $132''$; but he remarks: "*Situs relatiui observatio plurimis laborat difficultatibus,*" so that the difference of 12° between Herschel and d'Arrest is not surprising. Probably Sir John Herschel determined the R.A. and Decl. of the following one, measured the position-angle, estimated the distance, and afterwards from these data made out the $\Delta\alpha$ and $\Delta\delta$, in doing which he accidentally gave the latter the wrong sign, placing the preceding nebula south of the following one, instead of north of it.

II. $8-9 = h\ 316-317$.—M. Flammarion says that in 1830 (should be 1827) the position-angle was 30° to 40° ; while d'Arrest in 1862 (should be 1863-65) found it = $80^\circ \pm$. This is however not correct, for Herschel says: "Pos. by a drawing made at the time $30^\circ \dots 40^\circ$ *nf*," which means that he estimated it equal to $60^\circ \dots 50^\circ$. At Birr Castle the angle was in 1850 on two nights measured = 77° and $75^\circ.5$, and in 1876 = $78^\circ.8$. In 1783 Sir W. Herschel determined the places of these two nebulae, according to which the second one should then have been $1'$ south of the first one; but as his earliest observations are very inaccurate, no conclusion can be drawn from them. At any rate, the objects were stationary from 1850 to 1876.

Great Nebula in Orion.—This object has more frequently than any other nebula been suspected of having varied in form, but on the other hand it has been more thoroughly examined and discussed than any other. From his own observations and his

examination of the principal monographs, d'Arrest drew the conclusion that "the observed changes in this vast mass of gas seem exclusively to turn out to be temporary fluctuations of brightness,"* and the elaborate discussion of all previous observations in connection with his own led Professor Holden to state "that the figure of the nebula in *Orion* has remained the same from 1758 till now (if we except a change in its apex about 1770, which appears quite possible), but that in the brightness of its parts undoubted variations have taken place, and that such changes are even now going on."†

IV. 25 = *h* 428.—A double star, *h* 749, involved in (or projected on) a fan-shaped nebula. In 1827 the position-angle was estimated = 125° , and the distance $12''$; in 1863 d'Arrest estimated them 120° and $4''$. There has been no change, as two observations made at Birr Castle in 1874-76 give $119^\circ 11''$. But even if there had been a change in the distance of the double star, this could not really be called a change in a nebula. Sir W. Herschel did not notice the duplicity of the star.

II. 316-317 = *h* 444-445.—In the *Astr. Nachr.* No. 1366, d'Arrest called attention to the following remarkable discrepancy between the then existing observation of this fine double nebula:—

H	1785	Pos.	$^\circ$	Dist.	$60''$
<i>h</i>	1827	"	45	"	45
d'A	1862	"	56.5	"	28.5

At first sight this certainly looks like orbital motion. But, unfortunately, Sir John Herschel only estimated the angle and distance, and his two estimates of the latter differ very much *inter se*, being $30''$ and $60''$. It is, however, only fair to add that d'Arrest merely showed the disagreement without making it out to be a sure case of motion. The following later measures show that the pair have been at rest during the last twenty-five years:—

Schultz	1864-65	Pos.	$59^\circ 1'$	Dist.	$31'' 6$ (4 nights).
Dreyer	1876	"	$53^\circ 9'$	"	$27'' 7$ (Birr Castle, 1 night).
Dreyer	1887	"	$57^\circ 3'$	"	$32'' 3$ (2 nights).

The two nebulae are connected (see *Phil. Trans.* 1850, pl. xxxviii., and *Birr Obs.* 1848-78, pl. ii.). The following one is fainter, smaller, and far less condensed in the middle than the preceding one.

h 705.—This is a very interesting object: a double star (*h* 2529) with nebulosity attached. In "The Observatory," vol. viii., p. 127, Mr. H. Sadler suggested that here might be a possible case of proper motion in a nebula, since Mr. Burnham

* *Undersøgelse over de nebulose Stjerner* (1872), p. 42.

† Holden's *Monograph*, p. 225.

in 1879-82 saw the nebula "19" from the principal star," while Sir J. Herschel in 1830-31 said the nebula was about the principal star. It will be well to put together all the observations made by Herschel, both as they are recorded in his fifth list of double stars * and in the *Phil. Trans.* 1833.

Sweep.	Double Star Obs.		Obs. of Neb.
243	Pos. 95°3	Dist. 1 $\frac{1}{2}$ "	A most curious, delicate, and interesting object. The nucleus of a very faint nebula examined with 320, proves to be distinctly a first-class double star.
	" 10 8	" 7	
	" 95°4	" 1	A very close D * of the first class involved in a nebulous wisp. "A most curious, delicate, and interesting object."
242	" 8°0	" 7	
	" 330±†	" 6	
	" 7°0	" 18	A nebula strongly suspected about the close double star, and a fourth star also suspected.
338	" 7°0	" 18	
	" 7°0	" 18	A double star in a very faint nebula (a hurried observation).
	" 7°0	" 18	
	" 7°0	" 18	A triple star in a nebula, a fourth * suspected.
	" 7°0	" 18	
	" 7°0	" 18	A double * in a v F nebula.
	" 7°0	" 18	

In the "Remarks" on the double-star observations (*l.c.* p. 78) Herschel alludes to this object, saying that "a minute and very close double star forms the nucleus of a small round nebula; one or two other small stars in the immediate vicinity seem unconnected with it, but the exactly central position of the double star strongly points to a physical relation between them." It is, however, not to these remarks, written several years afterwards, but to the observations themselves that we must look for evidence of motion. D'Arrest observed the object three times in 1864-65; he saw a star of 10.11 magnitude with a star 13 mag. about 12" *nnf.*, the latter being the centre of a nebula which reached to the former star. In 1872 the following observation was made at Birr Castle: "Neb. to * 9m Pos. 193°0, Dist. 19"7, * 9 to * 12m 243°3, 8"8," and in 1876: "Double star, 10.11 and 15.16 mag. 5°0, 21", v F *neby* round it, E north and south." Mr. Burnham ‡ in 1879 and 1882 saw only the wide pair in Pos. 7°9, Dist. 19"0 (and once the faint companion *sp* the brighter star), and remarks, "The nebula is now 19" from the principal star."

All this does not seem to prove that the nebula has moved away from the close double star, for it is only in the *remarks* that Herschel calls the latter the nucleus of the nebula, while the observation in sweep 243 merely says that it was involved, which is not contradicted by the later observers. Mr. Burnham of

* *Mem. R.A.S.*, vi. p. 32.

† Should be 230°. See d'Arrest and Burnham.

‡ *Mem. R.A.S.* xlvii. p. 270.

course did not mean that the *edge* of the nebula was $19''$ from the close double, but that the nucleus was at that distance, which it is still. Having during the two years elapsed since Mr. Sadler's note appeared quite forgotten all the details about this object, I looked it up on March 15 last on finding it on my working list, and noted:—

"Nebulosity nearly reaches the south * if it does not actually touch it. It is oval in shape, E in the same pos. angle as the two stars. I saw only the two stars, but definition was not particularly good. The north star looks more like a nucleus than a *."

This agrees perfectly with d'Arrest's observations, so that there can have been no "proper motion" between 1865 and 1887, and it may safely be inferred that there has been none since 1830, unless we are to believe in this and similar cases that nebulae in the good old days moved about as they liked, but have been on their good behaviour since 1861 and kept quiet.

Great Nebula around η Argus.—It is sufficient to refer to the various papers on the alleged discovery of vast changes in this nebula in the *Monthly Notices*, vol. xxxi. There has ever since been perfect unanimity among astronomers that the changes were "altogether imaginary" (*Ibid.* xxxii. p. 178).

I. 248, II. 832 = *h* 983-984.—W. Herschel in 1790 made $\Delta\alpha = 12^s$, giving both nebulae same N.P.D. In 1832 J. Herschel found $\Delta\alpha = 12^s$, the second nebula being (estimated) $45''$ north of the first. In 1866 d'Arrest found $13^s.5$ and $57''$. "Mouvement certain," says M. Flammarion. On the 27th April last I found Pos. angle = $61^\circ.1$, Dist. = $120''.0$, or $\Delta\alpha = 14^s.0$, $\Delta\delta = 58''$. Both are pretty large and *vgb* M to a very soft-looking nucleus. No change.

III. 394-395 = *h* 1065-1067.—M. Flammarion considers that the position-angle has changed 20° since Sir J. Herschel in 1830 twice estimated it = 70° ; while d'Arrest in 1864 on three occasions states that they are on the same parallel. But in 1865 he says that the second one is "*pauillum quid ad boream*," and so it is, for at Birr Castle in 1872 it was measured in Pos. $82^\circ.7$, Dist. $69''.5$, and on April 11 last I found Pos. $79^\circ.5$, Dist. $68''.5$.

II. 751-752 = *h* 1905.—Two connected nebulae, the preceding one very little elongated, the following one much so; figured in *Phil. Trans.* 1833, fig. 77 (one observation) and 1861, fig. 31. The late Lord Rosse remarked (*l.c.* p. 704) that in Herschel's drawing the axes of the two nebulae are in a line; in 1850 Mr. G. Johnstone Stoney found them not to be in a line; in 1855 Mr. Mitchell (at Birr Castle) remarked that the axes were not in a line but were parallel; while in 1861 (when the drawing in P.T. 1861 was made) they were neither in a line nor parallel, but inclined at an angle of 16° . Since then the following observations have been made:—

Birr Castle, 1871. No E of *p* neb noticed, the axis of the *f* neb makes an angle of about 12° by a diagram.

Birr Castle, 1872. The *f* one E $130^{\circ}4$ (2 meas.), Pos. of line joining centres $117^{\circ}6$ (2). The E neb [*f* one] slightly cometic.

Armagh, 1887, April 27. *p* one vvl E, apparently towards *f* one, the latter very diffused and hazy, Pos. of E $128^{\circ}4$ (2 meas.), therefore difficult to measure; line joining them $120^{\circ}2$ (2 meas.).

The very hazy look of the two nebulae, the small amount of elongation of the first one, and the want of sharp condensation in both of them are more than sufficient to account for the disagreement between the various observers.

M $20=V$ 10, 11, $12=h$ 1991, 3718, the "trifid nebula." This forms the subject of an elaborate monograph by Professor Holden,* in which it is attempted to prove that from 1784 to 1833 the triple star was centrally situated between the three nebulosities, but that from 1839 to 1877 it was involved in the south following mass of nebulosity. The latter proposition rests on a firm basis, as the nebula has been repeatedly examined and drawn with every care since 1839; but this cannot be said about the first proposition. At the Cape of Good Hope, Sir J. Herschel made a drawing in a single night (August 1835), which exhibits the triple star on the very edge of the *sf* nebulosity. A careful drawing made at Slough was lost, and that engraved in P.T. 1883, fig. 80 (which shows the triple star in the midst of a vacuity) was constructed from sketches "the rudest imaginable aided by memory." The other evidence as to the position of the star between 1784 to 1833 consists in various notes by Sir William and Sir John Herschel. But the former never says that the double star was *in the middle of the vacuity*, but in 1784 he describes the object as "three nebulae faintly joined form a triangle; in the middle is a double star," and in 1786: "a double star with extensive nebulosity of different intensity; about the double star is a black opening." It is quite true that Sir John Herschel on three occasions† states that the star is in the middle of the vacuity. But is it so strange that at an altitude of only 15° , and during the strong twilight of our summer months (sweep 30 was made on July 1, 1826), the nebulosity could not be traced close to a bright double star of the 8.5 and 9th magnitude? It is at any rate curious that the critical time when the alleged motion of the nebula towards the star should have taken place is precisely the moment when we exchange strong twilight, very low altitude, rude sketches for little or no twilight, much higher altitude, and careful drawings.

The drawings made by Mason, Lassell, and Trouvelot differ in many details just as the various drawings of *Orion* do; and very possibly changes of brightness have taken place, both near

* *Am. Journ.* xiv. Dec. 1877.

† *Mem. R.A.S.* ii. p. 490, in a footnote to a paper on the *Orion* nebula, *ibid.* iii. p. 63, in his observations of double stars, and in sweep 30, *Phil. Tr.* 1833, p. 460.

the double star and in the other part of the nebula; but that the nebula should about 1835 in the course of a few years have *moved* so as to envelop the star, after which no sensible change occurred again so far as published observations go, does not seem sufficiently well proved.

M 17= h 2008, *Omega* nebula.—In the *American Journal*, vol. xi., May 1876, Professor Holden published a paper "On Supposed Changes in the Nebula M 17," in which he endeavours to show by a comparison of his own observations (made with the Washington 26-inch refractor) with those of J. Herschel, Lamont, Lassell and Trouvelot that the western branch of the Ω has moved relatively to the little group of stars 10, 3, 11 (Lassell) at the *np* end, and particularly with regard to the star No 1 at the *sp* side of the Ω .* The nebulosity is very diffused at the group of stars, yet there is very little difference between the various drawings, but the star 1, on which reliance is chiefly placed, was by J. Herschel in 1837, and Lassell in 1862, found to be on the inner (concave, north following) edge of the curve of the nebula; while Holden and Trouvelot in 1875 placed it well within the nebulosity, in fact preceding nine-tenths of it. A drawing by Le Sueur made in 1869 (*Proc. R. S.* vol. xviii., overlooked by Professor Holden) agrees with the one made at Washington, so that the change should have taken place between 1862 and 1869. But a drawing made by M. Tempel in 1876 with the 11-inch refractor at Arcetri† agrees with the *earlier* drawings in this particular, while two sketches made at Birr Castle in 1854‡ agree with the *later* drawings. There has therefore certainly not been any bodily shifting of the nebula, but the possibility of changes of brightness are not excluded.

II. 426-427= h 2087-2089.—M. Flammarion remarks that there is a great disagreement between J. Herschel's and d'Arrest's difference of declination, the former observer making $\Delta\delta=82''$, the latter $61''$. It should have been stated that it was only on one night that d'Arrest found $\Delta\delta=59''$ to $63''$, for the mean results of his three nights give $81''$. In 1876 I found at Birr Castle $78''\cdot4$ (one measure of pos. and distance).

III. 210-211= h 2202-2203.—M. Flammarion repeats a remark of d'Arrest's that J. Herschel in 1828 found $\Delta\alpha=10^{\circ}\cdot0$ (should be $9^{\circ}\cdot0$) and $\Delta\delta=37''$, while d'Arrest found $10^{\circ}\cdot5$ and $53''$. But the mean of d'Arrest's results gives $8^{\circ}\cdot2$ and $62''$. This faint pair, between which d'Arrest found a *v* F, *v* S nebula (G.C. 6112), and of which the elongation of the preceding one makes the measurement of $\Delta\delta$ difficult, has been frequently observed of late years, showing no appreciable change.

* See Mr. Lassell's drawing (fig. 32) and skeleton chart (fig. 32 A) in *Mem. R.A.S.* vol. xxxvi.

† See Winnecke's review in *Vierteljahrsschrift d. a. G.* xii. p. 245.

‡ *Observations of Nebulae*, 1848-78, pl. vi.

D'Arrest	1861-64	$\Delta\alpha = 8^{\circ}2$	$\Delta\delta = 62''$	3 nights.
Schönfeld	1861	9.72	59.5	5 "
Schultz	1863-65	9.05	61.0	3 "
Schönfeld	1864	9.38	71.3	1 night.
Vogel	1869	9.38	69.0	2 nights.
Dreyer	1877	8.92	60.7*	1 night.
Dreyer	1885	8.86	66.0	1 "

III. 855-856 = *h* 2294-2295.—These also occur in M. Flammarion's list, because W. Herschel in 1790 *estimated* the position-angle = 60° and the distance $60''$, while d'Arrest found 50° and $43''$. The objects are both excessively faint, and it would be easy to count up hundreds of similar discrepancies. In 1872 the angle and distance were measured at Birr Castle, and found = $51^{\circ}.0$ and $61''.5$.

I have spared no trouble in going through these cases one by one, although in some the evidence was of such a character as hardly to deserve a refutation. I would suggest to anybody who in future should feel inclined to lay a case of proper motion or variability of a nebula before the public, first to peruse the remarks of d'Arrest in the *Astr. Nachr.* vol. lvii. col. 342. In making micrometric observations of these interesting objects we must be content to work for unborn generations, or at least not to expect immediate and startling results, which would look well in popular books.

Armagh Observatory :

1887, *May*.

Note on the Effect of Refraction in Stellar Photography.

By J. L. E. Dreyer, Ph.D.

In his paper read at the April meeting of the Society, Mr. Grubb has assumed that a displacement of $0''.5$ is the smallest which would sensibly affect the symmetry of the image of a star on a photographic plate. It is of interest to see how soon refraction will move the image to this extent, assuming the action of the clock to be absolutely perfect and the instrument accurately adjusted †

The well known expressions for refraction in Right Ascension and Declination, first given by Bessel in the *Monatliche Correspondenz*, xvii. p. 214, are

$$57'' \frac{\tan t \sin \psi}{\cos \delta \sin (\psi + \delta)} \text{ and } 57'' \cot (\psi + \delta)$$

* In the single measure of distance at Birr Castle on Oct. 29, 1877, there is an obvious error of one revolution of the screw = $65''.1$.

† This question is not considered in Prof. Pickering's valuable paper, "Investigation in Stellar Photography."

where ψ is determined by $\tan \psi = \cos t \cot \phi$, while t is the hour-angle, δ the Declination, and ϕ the latitude. The rate of change of refraction is found by differentiation with regard to t .

$$\frac{da}{dt} = 57'' \left(\frac{\cos \psi}{\sin (\psi + \delta) \tan \phi} \right)^2 (1 + \tan \phi \tan \delta \cos t)$$

$$\frac{d\delta}{dt} = 57'' \left(\frac{\cos \psi}{\sin (\psi + \delta)} \right)^2 \cot \phi \sin t$$

The trail left on the plate after an exposure of n seconds of time will now be the resultant of the displacements along the parallel and the hour-circle equal to

$$\frac{15n}{206265} \frac{da}{dt} \cos \delta \quad \text{and} \quad \frac{15n}{206265} \frac{d\delta}{dt}$$

Assuming the latitude = 50° , the following table shows the values of these quantities for various parts of the sky for a change of 4^m , or 1° in hour-angle.

$t =$	0^h		1^h		2^h		3^h		4^h	
	$da \cos \delta$	$d\delta$	$da \cos \delta$	$d\delta$	$da \cos \delta$	$d\delta$	$da \cos \delta$	$d\delta$	$da \cos \delta$	$d\delta$
$\delta = 0^\circ$	0'99	0'00	1'07	0'33	1'33	0'79	1'99	1'68	3'98	4'10
+ 25°	0'71	0'00	0'73	0'16	0'81	0'36	0'96	0'64	1'26	1'12
+ 50°	0'64	0'00	0'64	0'13	0'66	0'28	0'69	0'45	0'72	0'67
+ 75°	0'71	0'00	0'70	0'16	0'66	0'31	0'60	0'47	0'51	0'63

The value of $da \cos \delta$ increases with south declination, being on the meridian and at $-10^\circ = 1''28$, and at $-20^\circ = 1''87$. Refraction accelerates the rising of a star, so that the apparent place of a star in the east will be in advance of the true one, but the difference of hour-angle gradually diminishes as we approach the meridian. After the culmination they change places, the apparent place now being behind the true one. A slight uniform retardation of the clock will thus do away with the greater part of the error in the direction of the parallel, as the table shows how very slowly $da \cos \delta$ changes with the hour-angle. The case is different with regard to the effect of refraction along the hour-circle, as the full amount of the change in this will cause displacement of the star on the plate, and it seems that a declination clock will ultimately become desirable, though the mechanical arrangements would be somewhat complicated if the instrument were to be available for lengthened exposures on any part of the sky. But this would be unnecessary in the case of systematic work, such as star-charting, as this could always be carried on close to the meridian, where the refraction in declination changes very slowly. Thus, for the latitude of 50° an equatorial star will be 27^m past the meridian, and a star of $+25^\circ$ Declination 39^m past the meridian, before a displacement of $0''5$ occurs.

The Right Ascensions of certain Stars within Ten Degrees of the Pole. Reduced from Observations by F. G. W. Struve.
By Henry Lefavour.

(Communicated by Prof. T. H. Safford.)

The observations from which the following star-places have been determined were made by Struve, at Dorpat in 1818 and 1819, and are published in the second volume of his "Dorpat Observations." The list includes all the stars that he observed which lie within ten degrees of the pole, with the exception of *Polaris*, the observations of which have already been reduced, and the stars which he designated as *Anonyma*, most of which are unimportant, and were observed but once, or at the most two or three times. The Right Ascensions are reduced to 1820, but no proper motions have been applied, and the epoch is consequently given for each star.

Catalogue of Mean Right Ascensions for 1820.

No.	Struve's Name.	Gr. No.	Misc. Cat. and No.	No. of Obs.	Mean Right Ascension for 1820.			Epoch.
					h	m	^s	
1	Cephei 320	67	...	3	0	19	46.097	1818.9
2	Ursæ Minoris 1	144	Bradley 65	9	0	39	10.712	1819.0
3	" Cephei 321	177	...	1	0	45	53.920	1818.9
4	Cephei 322	195	Bradley 95	1	0	48	21.260	1818.9
6	Rangiferi 15	424	...	3	1	47	55.007	1818.8
7	Rangiferi 16	426	...	2	1	48	33.430	1818.8
8	Rangiferi 21	505	...	1	2	12	33.640	1818.8
9	Rangiferi 23	527	...	2	2	22	35.215	1818.8
10	Rangiferi 26	580	Bradley 396	3	2	44	34.127	1818.9
11	Rangiferi 29	595	Bradley 402	2	2	51	33.490	1818.8
12	Rangiferi 323	642	...	4	3	8	58.663	1818.9

No.	Struve's Name.	Gr. No.	Misc. Cat. and No.	No. of Obs.	Mean Right Ascension for 1820 ^o .	Epoch.
					h m s	
13	Rangiferi 43	779	...	1	3 56 15 ^o 090	1818 9
14	Camelopardalis 36	856	Cephei 50 H.	1	4 37 7 ^o 100	1818 9
15	Camelopardalis 64	944	...	1	5 5 21 ^o 020	1819 5
16	Ursæ Minoris 4	1119	...	3	6 15 36 ^o 070	1819 1
17	Camelopardalis 120	1141	Cephei 51 H.	48	6 13 6 ^o 274	1818 9
18	Camelopardalis 131	1255	...	7	6 50 46 ^o 230	1818 8
19	Camelopardalis 132	1259	Camelopardalis 25 H.	3	6 52 38 ^o 480	1818 8
20	Camelopardalis 136	1278	...	6	7 1 24 ^o 485	1818 8
21	Camelopardalis 150	1339	Camelopardalis 28 H.	6	7 26 1 ^o 997	1818 8
22	Camelopardalis 152	1355	...	2	7 29 49 ^o 080	1818 8
23	Camelopardalis 156	1359	...	4	7 32 25 ^o 645	1818 8
24	Camelopardalis 170	1391	...	4	7 48 43 ^o 455	1818 8
25	Camelopardalis 180	1431	...	5	8 12 49 ^o 256	1818 8
26	Camelopardalis 182	1452	...	1	8 23 59 ^o 970	1818 9
27	Camelopardalis 183	1463	...	4	8 28 25 ^o 360	1818 9
28	Camelopardalis 184	1480	...	1	8 42 35 ^o 630	1818 9
29	Camelopardalis 186	1537	Draconis 1 H.	16	9 10 32 ^o 386	1818 9
30	Camelopardalis 189	1620	Camelopardalis 29 H.	4	10 1 44 ^o 428	1818 9
31	Camelopardalis 190	1633	Camelopardalis 30 H.	1	10 8 7 ^o 080	1818 9
32	Camelopardalis 191	1643	Bradley 1439	4	10 16 50 ^o 040	1818 9

No.	Struve's Name.	Gr. No.	Misc. Cat. and No.	No. of Obs.	Mean Right Ascension for 1820.	Epoch.
					h m s	
33	Camelopardalis 201	1778	...	2	11 16 46.300	1818.9
34	Camelopardalis 202	1782	...	3	11 18 42.923	1818.8
35	Camelopardalis 204	1845	...	2	11 50 39.380	1818.9
36	Camelopardalis 205	1850	...	3	11 55 27.077	1818.8
37	Camelopardalis 207 (pr.)	1858	Bradley 3241	4	12 2 41.960	1819.0
37 a	Camelopardalis 207 (sq.)	—	...	4	12 3 14.118	1819.0
38	Ursæ Minoris 5	1871	Bradley 1656	11	12 11 27.935	1818.9
39	Ursæ Minoris 6	1884	Bradley 1672	14	12 14 37.410	1818.9
40	Camelopardalis 209	1889	...	2	12 16 48.040	1818.9
42	Camelopardalis 211	1927	...	1	12 39 41.950	1818.9
43	o Camelopardalis 212	1940	Camelopardalis 32 H.	14	12 47 50.664	1819.0
44	Camelopardalis 213	1977	...	3	13 10 57.530	1818.9
45	Ursæ Minoris 12	2006	...	19	13 20 13.481	1819.0
46	Camelopardalis 214	2007	...	3	13 22 29.927	1818.8
47	Camelopardalis 216	2037	...	2	13 36 44.900	1818.8
48	Camelopardalis 219	2063	...	1	13 48 0.920	1818.9
49	Ursæ Minoris 20	2099	...	2	14 10 13.200	1818.9
50	Camelopardalis 223	2196	...	4	15 3 13.888	1818.9
51	Ursæ Minoris 45	2210	...	4	15 6 8.235	1818.9
52	Ursæ Minoris 44	2213	...	1	15 10 59.360	1818.9

No.	Starve's Name.	Gr. No.	Misc. Cat. and No.	No. of Obs.	Mean Right Ascension for 1820 o.	Epoch.
					^h ^m ^s	
53	1 π Ursæ Minoris (pr.)	2275	...	2	15 40 3'985	1819'4
53 ^a	1 π Ursæ Minoris (sq)	2276	...		15 40 16'765	1819'4
54	ϵ Ursæ Minoris	2422	...	70	17 4 45'175	1819'4
55	Cephei 4	2456	...	1	17 33 22'220	1818'8
56	Ursæ Minoris 79	2476	...	1	17 42 55'460	1819'0
57	δ Ursæ Minoris	2628	...	72	18 30 18'808	1819'3
58	24 Ursæ Minoris	2667	...	14	18 37 9'626	1818'8
59	prec. 75 Draconis	3268	Bradley 2701	2	20 37 45'315	1818'9
60	75 Draconis	3276	...	4	20 39 4'498	1818'9
61	74 Draconis	3277	...	2	20 39 26'350	1818'9
62	λ Ursæ Minoris	3308	...	14	20 37 26'841	1818'9
63	76 Draconis	3370	...	8	20 55 0'091	1818'9
64	Ursæ Minoris 86	3548	...	14	21 33 18'064	18 8'9
65	Cephei 180a prec.	3707	...	4	22 4 10'338	1818'9
65	Cephei 180a seq.	3709	...	4	22 4 16'853	1818'9
67	Cephei 221 prec.	3820	Cephei 32 H.	1	22 26 10'480	1818'8
68	Cephei 221a seq.	3824	Bradley 2997	1	22 26 48'360	1818'8
69	Cephei 233	3887	...	3	22 38 50'627	1818'8
70	Cephei 246	3928	Cephei 34 H.	1	22 47 54'780	1818'9
71	Cephei 253	3970	Cephei 36 H.	2	22 55 27'620	1818'8

On the Probable Errors of Transit-observing.

By W. H. Finlay, M.A.

In the winter months consecutive transits above and below pole of several stars are often secured on the same night, and a question arises as to how the separate results for azimuthal error are to be combined to form the "adopted" azimuthal error. To settle this question we require to know the probable errors of observation at different declinations, and the following determination of the probable errors of a single wire in transit-observing was undertaken for this purpose.

After my results had been obtained I became aware of the formula given by M. Struve ("Sur l'Emploi de l'Instrument des Passages"), but as the latitude of the Cape is very different from that of Pulkowa I have thought it well to give my results.

The following is the method I have adopted:—

Assuming the adopted wire intervals to be absolutely correct, the difference of the observed interval between two wires from the computed interval gives the actual error made in measuring a distance by observation at two wires; so that by taking the mean of a large number of such differences we get the "average" error of a distance measured by transit over two wires. If M be this "average" error, or mean of the differences without regard to sign, then

$$\text{probable error of a distance} = M \times [9.927046]$$

and

$$\text{probable error of a transit at one wire} = \frac{M}{\sqrt{2}} \times [9.927046].$$

The differences for each star were multiplied by $\cos$ (declination) to reduce to the equator, and the stars were grouped in zones according to their North Polar distances. The observations which I have used were made with the transit-circle of the Royal Observatory, Cape of Good Hope, by the three observers who took the chief part in the observations for the Cape Catalogue, 1880; but the period which I found most convenient for my purpose was 1879–1881; the power of the eyepiece used was 180.

The following table gives the probable errors for different N.P.D.'s of an observation at one wire as observed, and also when reduced to their equivalents at the equator:—

TABLE I.

N.P.D. of Zone.	Mean Z.D. of Zone.	F. p.e. at Obs.	No. of Obs.	M. p.e. at Obs.	No. of Obs.	P. p.e. at Obs.	No. of Obs.	F.	Reduced p.e. M.	p.e. P.
80-100	34	0.049	208	0.086	380	0.063	872	0.049	0.086	0.063
100-123	13	0.067	271	0.094	362	0.070	444	0.063	0.088	0.066
124-135	6	0.061	163	0.084	148	0.078	171	0.047	0.064	0.060
135-150	18	0.067	239	0.092	269	0.080	221	0.041	0.057	0.050
150-165	33	0.081	263	0.110	266	0.101	318	0.031	0.042	0.040
166-170	44			0.203	159	0.168	194		0.042	0.035
171-174	48½			0.278	155	0.258	279		0.036	0.034
175-177	52			0.483	340	0.410	626		0.034	0.029
178	54			0.891	120	0.736	115		0.031	0.026
179	55			1.408	88	1.129	125		0.025	0.020
181	57			1.367	60	1.229	97		0.024	0.022
182	58			0.789	26	0.845	110		0.028	0.029
183-185	60			0.553	443	0.475	546		0.039	0.033
186-189	63½			0.260	42	0.302	200		0.034	0.039
190-194	68			0.317	111	0.227	216		0.066	0.047

There were not sufficient observations of circumpolar stars by F in the period 1879-1881 to give a determination of his probable errors beyond 165° N.P.D., and the series for M does not rest on very large numbers of observations except for the zones 175-177 and 183-185. I shall therefore take P as the average observer, and any further conclusions will be based on the figures in the last column of Table I. It will be seen that the probable errors of a single wire by the different observers, though differing in amount, follow the same general law; and that the probable error of observing does not by any means increase in an inverse ratio to the cosine of the star's declination.

If we try to represent our numbers by the formula

$$P.E.^2 = a^2 + b^2 \sec z \sec^2 \delta,$$

we find

$$a^2 = .003978, \quad b^2 = .000564,$$

and therefore

$$P.E. = \sqrt{(.003978)^2 + (.000564)^2 \sec z \sec^2 \delta}.$$

The representation of the probable errors by this formula, and some variations of it which I have tried, is, however, only moderately satisfactory. The formulæ all failed to give the rapidly diminishing probable errors near the pole. Nor am I surprised at this failure; for, at a distance of 2° or 3° from the pole, the star's disc is in contact with the wire for several seconds,

and the observer practically alters his method of observing: he fixes his attention on the wire and waits till the star's disc is equally distributed on the two sides of it, and this observation can be made with considerable certainty and accuracy.

To find the error in the determination of the azimuth of the instrument introduced by error in observing, divide these probable errors by $\sin$ (zenith distance of group), and we get the following table of probable errors of azimuth reduced from a single wire:—

TABLE II.

N.P.D. of Star.	p.e. of azimuth.	Combination Weight.	Approx. Weight.
168 ⁰	^s 0'050	2'45	5
172 $\frac{1}{2}$	0'045	3'16	6
176	0'036	4'57	9
178	0'032	5'57	11
179	0'024	10'00	20
181	0'026		20
182	0'035		11
184	0'038		9
187 $\frac{1}{2}$	0'044		6
192	0'051		5

so that *four* wires of a star about $178\frac{1}{2}^{\circ}$ N.P.D. are of as much value in determining the azimuthal error as *sixteen* of a star at 170° N.P.D.

We learn another fact from the above table; viz. that at the latitude of the Cape the probable error of the deduced azimuth is the same whether the star be observed above or below the pole; that is to say, the increase of azimuth-factor for the star below pole is counterbalanced by the increased error of observation due to inferior definition at the greater zenith-distance.

The last column in Table II. gives approximate weights for combining azimuthal errors determined from stars at various N.P.D.'s.

Royal Observatory, Cape of Good Hope:

1887, March 31.

Notes on a MS. Eclipse Volume. By the Rev. S. J. Johnson, M.A.

The accompanying MS. volume * gives eclipses in this country for a period of about 2,000 years, from A.D. 538 to A.D. 2500, being recorded ones of both luminaries from the date of the first in 538 to 1200; all solar eclipses visible here from A.D. 1200 to A.D. 2200, omitting a very few in which scarcely a tenth of the Sun's diameter is obscured, including lunar ones for a certain period and large solar eclipses from A.D. 2200 to A.D. 2500.

It will be seen that during the 1000 years from A.D. 1200 to A.D. 2200 there are only eight cases of an interval so long as that through which we are now passing (May 1882 to June 1890) without a solar eclipse visible in England generally.

The only solar eclipses total at Greenwich in the period of 2000 years are those of 878 and 1715, as has been before remarked, unless, perhaps, that of 664 is an instance. Between the eclipse of 1715 and the preceding one that most nearly approached the total phase at Greenwich, viz. that of 1140, there is an interval of 575 years and a month. A similar interval reckoned forwards from 1715 brings us to A.D. 2290, when there is an eclipse soon after sunrise that seems total in the extreme north of the kingdom. A similar interval reckoned previously to 1140 brings us back to February 565, when the Sun appears to have been totally eclipsed across this country just before setting. Though no ancient chroniclers appear to record this directly, yet Matthew of Westminster says, "In this year there were seen many signs in the Sun and Moon."

Melplash Vicarage, Dorset:
April 28.

* This volume is placed in the Library.

Observations of Saturn and δ Geminorum, Jan.-Feb., 1887. By John Tebbutt.

The accompanying positions of *Saturn* have been deduced from comparisons of the planet with the clock star δ *Geminorum* about the time of their conjunction. The measures were made with an excellent position flar micrometer on the Grubb 8-inch equatorial refractor, transits of the planet's western limb and the star being observed by means of a half-second chronometer across the single position thread in the meridian, and differences of Declination obtained between the star and the planet's south limb. The micrometer screws have been well tested, one revolution of the screw employed being 17".869. The differential Right Ascension and Declination have been corrected for refraction. Corrections due to the planet's semi-diameter (see *Nautical Almanac*, page 282) have been applied to these coordinates and to the times of transit of the limb. The apparent place of the star has been adopted from the *Nautical Almanac*, and the places of the planet have been finally corrected for parallax. As the definition and steadiness of the images were all that could be desired, I have no doubt that the concluded positions will bear favourable comparison with those derived from meridian observations. The last two columns of the table subjoined exhibit a comparison of the results with the places interpolated from the ephemeris on page 258 of the *Nautical Almanac*.

1887. Jan. 30 Feb. 1 " 8 " 9 " 14	d	Windsor Mean Time.			Planet's Centre—Star $\Delta\alpha$			Comp.	Planet's Geocentric Apparent δ			Obs.—N.A.	
		h	m	s	m	s	"		h	m	s	α	δ
		10	35	10	+1	58	32	9	22	10	52.1	-0.10	-1.9
		10	3	46	+1	22.15	+1	0.3	22	12	11.8	-0.11	-0.7
		9	37	38	-0	36.77	+5	18.3	22	16	30.0	-0.19	-0.5
		10	9	37	-0	52.91	+5	53.3	22	17	5.1	-0.18	-0.5
		10	28	54	-2	6.86	+8	34.9	22	19	46.9	-0.19	-0.5

Windsor, N.S. Wales: 1887, March 12.

Occultation of α Tauri. 1887, March 2. By G. L. Tupman.

Notwithstanding the daylight, the Moon's dark limb was plainly visible. *Aldebaran* was very bright and flaring, and hung a long time on the limb without being projected on it more than half of its apparent diameter, which was $10''$ or $15''$, the reddish colour being very remarkable. By the click of the armature of the transit-clock-relay, at 50° the star suddenly waned, but remained conspicuous until 51° , the light lingering, at less than one-half of its original intensity, for fully one second of time. The point of disappearance was on the dark limb, well clear of the terminator.

Having placed a wire tangent to the Moon's bright limb at the calculated point of reappearance, the star was seen to re-appear instantaneously. It was very much less bright at the bright limb than it had before appeared. Watched for 30 seconds there was no further increase of its light. It hung on the limb and was fairly bisected by the latter for a considerable time.

Achromatic of $4\frac{1}{2}$ inches aperture, power 66.

	Local Sid. Time.			G.M.T.		
	h	m	s	h	m	s
Disapp.	4	28	39.5*	5	49	11.6
Reapp.	4	40	33.4	6	1	3.6

The occultation was observed by Mr. E. B. Knobel at Bocking, who remarks:—

"The star *glided out* (at the dark limb near the terminator). It did not *snap out* as usual, but there was an appreciable time in its disappearance; it was not an instantaneous phenomenon. The observation was good.

"The reappearance (at the bright limb) seemed instantaneous, but the observation was not quite so good as the immersion. Time between observations 9 minutes 30 seconds."

Sextant Observations of Comet α , 1887, on board the ship "British Merchant," By Captain E. J. Molony.

(Communicated by Captain H. Toynbee.)

1887, January 21.—Lat. $28^\circ 17'$ S. Long. $21^\circ 25'$ W.

At 8 P.M. saw for the first time a comet; a long slightly curved tail, originating in a point towards the Sun, and stretching for $18\frac{1}{2}^\circ$ towards the nearest (smallest) Magellan Cloud, from which its visible termination is distant 18° . As the comet is very ill defined and sky cloudy the following distances are approximate. G.M.T. $10^h 11^m$:—

* Corresponding to the recorded 50° by the clock.

Canopus from head of Comet	...	...	75° 10'
α Crucis " " "	...	...	67 58
Rigel " " "	...	...	104 48

January 22.—Lat. $27^{\circ} 36'$ S. Long. $21^{\circ} 44'$ W. G.M.T.
 $10^h 27^m$. Comet more indistinct. Approximate distances of point:—

Canopus	...	..	...	73° 50'
α Crucis	...	...	...	66 38
α Arietis	...	...	...	91 48

January 25.—Lat. $23^{\circ} 4'$ S. Long. $24^{\circ} 27'$ W. G.M.T.
 $10^h 37^m$. Comet visible but faint. Approximate distances:—

From Canopus	...	...	...	66° 10'
,, α Crucis	...	...	...	66 0
,, Rigel	...	...	...	90 30

On the Inclinations of Cometary Orbits. By W. H. S. Monck.

Mr. Chambers in his *Handbook of Descriptive Astronomy* has tabulated the computed cometary orbits up to the year 1876, in which year the last edition of his well-known book was published. He arrives at the conclusion that there is "a decided disposition in the orbits to congregate in and around a plane inclined 50° to the ecliptic." His figures, I think, hardly bear out this result, though on dividing the inclinations into intervals of 10° the largest number appears in the interval between 40° and 50° . But the comets whose orbits are inclined at angles of 70° to 90° amount to sixty-seven, while those whose orbits are inclined at angles of 10° to 30° number only forty-three, both being equally removed from the supposed median plane at 50° .

It occurred to me to examine the orbits of comets which appeared since the publication of Mr. Chambers's List, and I found that the tendency towards orbits inclined at angles of over 50° became much more marked. Many of the earlier orbits being unreliable, however, I thought it best, instead of simply adding the later comets to Mr. Chambers's Table, to tabulate the inclinations of those which have appeared since A.D. 1650, omitting all which are known or believed to be identical. Such a table of course involves some uncertainties, but I believe the following will be found substantially correct:—

Inclinations of Cometary Orbits to the Ecliptic.

(Containing only Comets which appeared since A.D. 1650.)

0°	to	10°	...	...	20	per cent.
						7.7
10	„	20	...	...	19	7.3
20	„	30	...	...	15	5.8
30	„	40	...	...	30	11.5
40	„	50	...	...	37	14.2
50	„	60	...	...	33	12.7
60	„	70	...	...	32	12.3
70	„	80	...	...	37	14.2
80	„	90	...	...	37	14.2
Total					260	99.9

Here the tendency to a maximum at 50° has entirely disappeared. There are in fact more comets in the interval 70° to 90° than in the interval 40° to 60°. But on the other hand there is a very marked deficiency in inclinations from 0° to 30°. In the interval from 30° to 40° we first reach the average number. From 40° to 90° the number is invariably in excess of the average.

If we were at liberty to distinguish comets of short period (which are now known to be pretty numerous) from those which move in very elongated ellipses, parabolas, or hyperbolas, the results would be more striking. Comets of short period have even a more decided preference for small inclinations than the other comets have for large ones. And even if we do not adopt Mr. Proctor's theory, that these comets of short period have been ejected from the planets (especially *Jupiter*), or Professor Kirkwood's suggestion that some at least of them have a common origin with the asteroids, I think there is considerable reason for regarding them as original members of the solar system, exhibiting many of the characteristic properties of that system. If so, it is only to the other and larger class of comets that we can look for information as to the prevailing direction of motions in exterior space; for that their high inclinations to the ecliptic is not caused by anything in the solar system is, I think, certain. A comparison of the inclinations of their orbits with those computed for binary stars would be interesting from this point of view.

On the Orbit of Comet II. 1883. By Robert Bryant, B.A., B.Sc.

In the *Monthly Notices* for last November Lieut.-General Tennant gave parabolic elements of the orbit of this comet, and expressed his opinion that there is no justification for assigning an orbit with an eccentricity different from unity.

The elliptic elements, which I communicated to the Society in June 1885, depended upon three observations which I considered fairly good ones, and from these the orbit was deduced with great care. The difference between this orbit and that of Lieut.-General Tennant caused me to revise my work and to take into consideration a larger number of observations. Moreover the large residuals given in the middle observed place by Lieut.-General Tennant's parabola seemed to me to indicate that this latter curve scarcely sufficed to represent the observations, and further investigation has confirmed this idea.

As the result of a threefold interpolation the residuals given below were obtained by comparing the observations with the following approximate parabolic elements referred to the mean equator of 1884.0.

T	1883, Dec. 25 ^d 30154, G.M.T.
ω'	113° 36' 50".5
Ω'	254 33 46.1
i'	110 37 51.1
$\log q$	9.490993.

Observation—Computation.

	R.A. s	δ "	
1884, Jan. 12	+3.17	-21.0	Melbourne
17	+1.2	+85.0	"
18	+3.50	+1.0	"
19	+2.43	-1.8	Windsor, N.S.W.
21	+1.65	+4.2	"
21	+2.05	+1.1	"
22	+1.87	-0.2	"
22	+2.50	-1.1	Madras.
23	+1.89	-2.3	Windsor.
23	+2.13	-18.6	Madras.
24	+2.28	+2.7	Windsor.
25	+1.76	-8.9	"
25	+2.06	+6.1	Madras.
26	+2.61	+23.0	"
26	+3.55	-3.2	"
27	+1.07	-8.3	Windsor.
28	+2.11	-12.4	Melbourne.
28	+1.96	-0.1	Windsor.
28	+2.37	-1.8	Madras.
29	+1.90	-14.7	Melbourne.
30	+3.26	+19.0	Madras.
31	+0.19	+2.8	"
Feb. 1	-0.25	+5.1	Melbourne.
2	+2.13	-7.8	Windsor.
4	+2.47	+0.3	Melbourne.

The observations of January 17 and February 1 were rejected for discordance. The observation of January 29 was rejected in order to make the time of the second normal fall at a suitable epoch.

Taking the mean of Tebbutt's observations on January 21, a normal was formed from the observations up to January 22.

Taking the mean of the observations on January 26 and January 28, the second normal was formed from the observations from January 23 to January 28.

Taking the mean of the Madras observations on January 30 and 31, and assigning half weight to the result, the third normal was formed from the remaining observations.

The resulting normals referred to the mean equinox of 1884 January 0.0 are—

	R.A.	δ	
(1)	$343^{\circ} 41' 0''$	$-41^{\circ} 47' 16''.7$	for 1884 Jan. 19.0
(2)	$350^{\circ} 40' 57''.7$	$-41^{\circ} 59' 21''.7$	„ 25.0
(3)	$357^{\circ} 13' 1''.3$	$-41^{\circ} 44' 16''.7$	Feb. 2.0

From these normals the elements finally deduced are—

T	1883, Dec. 25.233782	δ
ω'	$113^{\circ} 16' 48''.74$	} Mean Equator, 1884.0.
Ω'	$254^{\circ} 32' 16''.97$	
i'	$110^{\circ} 31' 34''.08$	
e	0.9944387	
$\log q$	9.4901604	
P	414 years.	

Of course any period deduced from an arc of anomaly of only 14° and from an interval of only 14 days is deserving of little confidence.

These elements give the following residuals in the three normal places:—

Observation—Computation.

$\cos \delta$ dR.A.	-0.6	-1.3	+2.8
$d\delta$	0.0	-0.3	-0.1

Referred to the ecliptic of 1884.0 the above elements become—

T	1883, Dec. 25.233782, G.M.T.	δ
ω	$138^{\circ} 17' 59''.05$	} Mean Ecliptic, 1884.0.
Ω	$264^{\circ} 20' 17''.59$	
i	$114^{\circ} 54' 0''.64$	
e	0.9944387	
$\log q$	9.4901604.	

The parabolic elements given by Lieut.-General Tennant on p. 26, vol. xlvii. of the *Monthly Notices*, give the following residuals for the above three normal places :—

Observation—Computation.

Cos δ R.A.	— 4''8	— 85''7	— 141''4
$\delta\delta$	+ 612'5	+ 656'7	+ 685'3

Note.—When the above results were communicated to the Society, I was not in possession of the correction to the inclination of Lieut.-General Tennant's parabola given on p. 394 of the present volume. With this correction the errors (observation—computation) of the parabolic elements are respectively—

Cos δ R.A.	— 17''1	— 13''2	+ 18''2
$\delta\delta$	— 7'3	— 6'5	— 2'9

I think the orbit is undoubtedly elliptic, but what the eccentricity is it is impossible to state with certainty.

The Melbourne observations referred to above are those given in the *Astronomische Nachrichten*, and were published after my former paper on this subject was communicated to the Society.

The Orbits of Comets Fabry and Barnard-Hartwig. By J. Morrison, M.D., Ph.D., Assistant on the American Ephemeris, and Professor of Chemistry in the National University, Washington.

Comet Fabry.

The observations upon which the following hyperbolic elements of this comet are founded are as follow :—

Greenwich M.T.		Appar. α	Appar. δ
	d	h m s	
1885, Dec.	7'536032	0 24 46'01	+ 20° 52' 34''9
1886, Mar.	7'316539	23 19 42'92	+ 31 16 44'8*
„ April	1'339713	23 18 30'86	+ 38 37 57'0
„ June	6'953118	8 47 52'15	— 40 38 0'5

The first is a meridian observation made at Ann Arbor; the second results from extra meridian observations made at Greenwich and Paris, the comparison star being the same at both places; the third was made at Bothkamp and was obtained from *Astronomische Nachrichten*, No. 2703; and the fourth was made at Sydney, the comet being at the time “extremely faint, but in a good position for observation with Cape Cat. (1880) 4707,” fifty comparisons having been made. (*Monthly Notices*, vol. xlv. p. 496.) These observations were corrected for aberration and parallax by means of approximate parabolic elements.

* Geocentric.

T	1886, April 5 ^d 9520 Greenwich M.T.
ω	126° 34' 49" 215
Ω	36 22 11.454 Mean Equinox, 1886.0
i	82 37 6.012
e	1.00047857
$\log a$	3.1278354
$\log q$	9.8077809

The two middle places are well represented.

The interval between the extreme observations is 181.42 days, during which the comet described 208° 9' of its orbit.

Comet Barnard-Hartwig. 1886.

From the following observations the first and third of which were made at Washington, D.C., and the second at Kiel (*Astronomische Nachrichten*, 2753), an elliptic orbit of long period is obtained, the elements of which are given below:—

Greenwich M.T.	Appar. a.	Appar. δ .
1886, Oct. 7 ^d 918987	h m s 10 42 9.32	+ 1° 22' 6" 3
" " 29 708660	11 39 22.82	+ 5 49 19.4
" Dec. 2 976975	15 32 3.91	+ 17 58 55.8

By means of approximate parabolic elements computed from a shorter interval, the corrections for aberration and parallax were obtained and applied.

T	1886, Dec. 16 ^d 514158 Greenwich M.T.
ω	86° 21' 58" 570
Ω	137 21 36.163 Mean Equinox, 1886.0
i	78 22 25.525
e	0.99872521
$\log a$	2.7162151
$\log q$	9.8216538

Motion retrograde. The middle place is exactly represented. These elements give a period of 11,866 years, which is of course very uncertain, since the interval between the extreme observations is far too short to determine this element accurately in an orbit such as this is.

The formulæ for the equatorial rectangular coordinates are—

$$\begin{aligned}
 x &= [9.8740086] r \sin(v + 6^\circ 52' 45" 171) \\
 y &= [9.8268980] r \sin(v + 198 \quad 35 \quad 9.651) \\
 z &= [9.9977333] r \sin(v + 102 \quad 5 \quad 28.657)
 \end{aligned}$$

Ephemerides of the Satellites of Mars during the Oppositions of 1888 and 1890. By J. Morrison, M.D., M.A., Ph.D., Assistant on the American Ephemeris, and Professor of Chemistry, National University, Washington.

These ephemerides have been computed from the following elements of the orbits of the satellites, referred to the equator and equinox of the respective epochs:—

Phobos.

Epochs	1888, April 11 ^o	1890, May 27 ^o Greenwich M.T.
Period	^d 0 ^h 3189113 (mean solar)	
μ	1128 ^o 8405	
a	12 ^{''} 953 (at distance unity)	
i	36 ^o 44 ['] 6	36 ^o 44 ['] 0
N	47 18 ^o 2	47 19 ^o 0
u	17 40	117 53 ^o 6

Deimos.

Epochs	1888, April 11 ^o	1890, May 27 ^o Greenwich M.T.
Period	^d 1 ^h 262435 (mean solar)	
μ	285 ^o 16322	
a	32 ^{''} 354 (at distance unity)	
i	35 ^o 36 [']	35 ^o 35 ['] 5
N	48 10 ^o 66	48 11 ^o 5
u	225 41	172 21 ^o 5

*Greenwich Mean Time of Greatest Elongation.**Phobos.*

G.M.T.				p	α	δ	G.M.T.				p	α	δ
1888. d	h	m					1888. d	h	m				
Mar. 21	7	53 ^o 5	W	306 ^o 4	19 ^{''} 1	6 ^{''} 1	Mar. 30	6	8 ^o 5	W	o	"	"
22	10	40 ^o 4	E				31	8	55 ^o 3	E			
23	13	27 ^o 3	W				Apr. 1	11	42 ^o 1	W	305 ^o 8	20 ^o 5	6 ^o 8
24	16	14 ^o 2	E				2	14	29 ^o 0	E			
25	19	1 ^o 1	W				3	17	15 ^o 8	W			
26	21	48 ^o 0	E	126 ^o 2	19 ^o 6	6 ^o 4	4	20	2 ^o 6	E			
28	0	34 ^o 8	W				5	22	49 ^o 4	W			
29	3	21 ^o 7	E				7	1	36 ^o 1	E	125 ^o 3	21 ^o 0	7 ^o 3

G.M.T.				<i>p</i>	<i>a</i>	<i>b</i>	G.M.T.				<i>p</i>	<i>a</i>	<i>b</i>
1888. d	h	m		°	"	"	1888. d	h	m		°	"	"
Apr. 8	4	22.8	W				Apr. 24	22	4.4	E			
9	7	9.6	E				26	0	51.3	W			
10	9	56.3	W				27	3	38.2	E			
11	12	43.1	E				28	6	25.1	W			
12	15	29.8	W	304.9	21.3	7.6	29	9	11.9	E	123.3	21.0	8.0
13	18	16.6	E				30	11	58.8	W			
14	21	3.3	W				May 1	14	45.8	E			
15	23	50.1	E				2	17	32.7	W			
17	2	36.9	W				3	20	19.7	E			
18	5	23.7	E	124.4	21.4	7.8	4	23	6.7	W	302.7	20.5	8.1
19	8	10.5	W				6	1	53.8	E			
20	10	57.2	E				7	4	40.9	W			
21	13	44.0	W				8	7	28.0	E			
22	16	30.8	E				9	10	17.1	W			
23	19	17.6	W	303.7	21.2	8.0	10	13	4.3	E	123.0	20.0	8.2

Deimos.

G.M.T.				<i>p</i>	<i>a</i>	<i>b</i>	G.M.T.				<i>p</i>	<i>a</i>	<i>b</i>
1888. d	h	m		°	"	"	1888. d	h	m		°	"	"
Mar. 20	15	43.4	E	125.1	47.8	15.5	Apr. 23	16	47.0	E			
22	13	7.9	W				25	14	10.7	W			
24	10	32.3	E				27	11	34.4	E	122.1	52.6	20.1
26	7	56.5	W				29	8	58.3	W			
28	5	20.5	E				May 1	6	22.4	E			
30	2	44.4	W	304.6	50.4	16.8	3	3	46.9	W			
Apr. 1	0	8.1	E				5	1	11.9	E			
2	21	31.6	W				6	22	37.2	W	301.4	51.2	20.1
4	18	54.9	E				8	20	2.8	E			
6	16	18.2	W				10	17	28.6	W			
8	13	41.4	E	124.0	52.6	18.4	12	14	54.6	E			
10	11	4.5	W				14	12	20.8	W			
12	8	27.6	E				16	9	47.1	E	121.0	50.6	20.0
14	5	50.7	W				18	7	13.4	W			
16	3	13.8	E				20	4	39.8	E			
18	0	37.0	W	302.9	53.5	19.6	22	2	6.4	W			
19	22	0.3	E				23	23	33.1	E			
21	19	23.6	W				25	21	0.0	W	300.2	48.8	19.7

Phobos.

1890.				G.M.T.			1890.			G.M.T.			<i>p</i>			<i>a</i>			<i>b</i>			
				d	h	m				d	h	m				°	"	"				
May				8	0	44.8	E	123.6	23.5	2.3	June				7	3	52.1	W				
				9	3	31.8	W								8	6	39.0	E				
				10	6	18.8	E								9	9	26.0	W				
				11	9	5.8	W								10	12	12.9	E	126.7	26.5	5.3	
				12	11	52.8	E								11	14	59.9	W				
				13	14	39.8	W	304.1	25.0	3.0					12	17	46.8	E				
				14	17	26.8	E								13	20	33.8	W				
				15	20	13.8	W								14	23	20.8	E				
				16	23	0.8	E								16	2	7.7	W	307.1	26.0	5.6	
				18	1	47.8	W								17	4	54.8	E				
				19	4	34.7	E	124.5	25.4	3.3					18	7	41.7	W				
				20	7	21.7	W								19	10	28.9	E				
				21	10	8.6	E								20	13	16.0	W				
				22	12	55.6	W								21	16	3.2	E	127.3	25.5	5.7	
				23	15	42.5	E								22	18	50.2	W				
				24	18	29.5	W	305.0	26.0	3.8					23	21	37.4	E				
				25	21	16.4	E								25	0	24.6	W				
				27	0	3.3	W								26	3	11.9	E				
				28	2	50.2	E								27	5	59.2	W	307.5	24.9	5.7	
				29	5	37.0	W								28	8	46.4	E				
				30	8	23.9	E	125.6	26.3	4.3					29	11	33.7	W				
				31	11	10.8	W								30	14	21.0	E				
June				1	13	57.6	E				July				1	17	8.3	W				
				2	16	44.6	W								2	19	55.6	E	127.7	24.1	5.7	
				3	19	31.4	E								3	22	42.9	W				
				4	22	18.4	W	306.1	26.7	4.8					5	1	30.2	E				
				6	1	5.2	E								6	4	17.6	W				

Deimos.

1890.	G.M.T.			<i>p</i>	<i>a</i>	<i>b</i>	1890.	G.M.T.			<i>p</i>	<i>a</i>	<i>b</i>
	d	h	m					d	h	m			
Apr.	30	22	40.9	E	122.5	55.1	5.6	May	16	2	3.6	E	
May	2	20	6.9	W				17	23	28.0	W		
	4	17	32.7	E				19	20	52.3	E	124.0	63.7 9.4
	6	14	58.3	W				21	18	16.6	W		
	8	12	23.6	E				23	15	40.9	E		
	10	9	48.9	W	303.1	59.7	7.2	25	13	5.1	W		
	12	7	14.0	E				27	10	29.1	E		
	14	4	38.9	W				29	7	53.1	W	304.9	66.1 11.8

G.M.T.				<i>p</i>	<i>a</i>	<i>b</i>	G.M.T.				<i>p</i>	<i>a</i>	<i>b</i>					
1890.	d	h	m	°	"	"	1890.	d	h	m	°	"	"					
May	31	5	17	1	E		June	19	3	21	4	E						
June	2	2	41	1	W			21	0	46	7	W						
	4	0	5	3	E			22	22	12	3	E						
	5	21	29	4	W			24	19	38	1	W						
	7	18	53	6	E	125	6	26	17	4	1	E	126	5	62	1	15	2
	9	16	17	9	W			28	14	30	3	W						
	11	13	42	4	E			30	11	56	6	E						
	13	11	6	9	W		July	2	9	23	1	W						
	15	8	31	5	E			4	6	49	8	E						
	17	5	56	3	W	306	1	6	4	16	8	W	306	6	57	6	14	5

In these ephemerides p denotes the position-angle of the major axis of the satellite's apparent orbit, and is reckoned from the north towards the east, and from 0° to 360° ; a and b denote the major and minor semi-axes of the apparent orbit. During these two oppositions of the planet the satellites move in the direction of *increasing* position-angles, the Earth being *above* the plane of the orbits. The time of greatest elongation has been given to the nearest tenth of a minute in order that a sufficiently accurate comparison may be had with observation.